16.06 Lecture 7
Transient Response Characteristics and System Stability

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Today's Topics

1. Graphical determination of residues
2. System stability
3. Simple lag system
4. Quadratic lag system

Reading: 4.4

1 Graphical determination of residues (complex conjugate poles) (vdv 1.8, 1.9)

(a) The technique for real poles (Lecture 7) applies here, but the residues are complex conjugates.

(b) We consider

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \tag{1}$$

The symbols ζ and ω_n will take on physical meaning shortly.

(c) For $\zeta < 1$, the poles of Eq. (1) are

$$\begin{aligned} s_1 &= \\ s_2 &= \\ s_3 &= \end{aligned}$$

The pole-zero pattern is:

The PFE is:

$$C(s) = \tag{2}$$

(d) Why do we consider $\zeta < 1$ (underdamped)?

Well:

when $\zeta = 1$ (critically damped):

when $\zeta > 1$ (overdamped):

In either case, the poles are on the negative real axis. This is not typical for aero/astro systems.

(e) Eq. (2) shows that the time response will be:

If you like this stuff, please read pgs. 23-25 of the text.

However, Prof. Markey recommends the following shortcut...

(f) For a complex conjugate pair, use

Example 1.9.4 (see next page):

Let us make sure we understand how to apply the above rule.

insert Example 1.9.4 here

2 Transient response characteristics and system stability (vdv 4.4)

2.1 Introduction

(a) Given $C(s) = G(s)R(s)$

the poles of $R(s)$

the poles of $G(s)$

For now, we will focus on the transient solution.

(b) We know that the system poles are either

or

The transient solution is the sum of the responses from these two types.

(c) We will focus on

2.2 System stability

(a) Definitions:

2.3 Simple lag (first-order system)

(a) Step response:

$$C(s) =$$

T is the time constant. It is the time in seconds for the decaying exponential transient to be reduced to $e^{-1} = 0.368$ of the initial value.

See Figure 4.10 (on the next page)

insert fig 4.10 here

(b) Stability and speed of response:

The system is stable if

The speed of response increases as

(c) Physical examples:

2.4 Quadratic lag (second-order system)

(a) Step response:

$$C(s) =$$

This expression uses the standard form for a second-order system.

ω_n is

ζ is

We now know that for $\zeta < 1$

$$c(t) =$$

where $\omega_d =$

$\theta =$

(b) Second-order poles

(c) Second-order system time constant is the time in seconds for the amplitude of the oscillation to decay to e^{-1} of its initial value:

$$T =$$

At $t = 4T$, the amplitude of the transient has decayed to 2% of the initial value. See Fig 4.13.

(d) Pole position and dynamic behavior (see top of pg. 111 in text).

(e) Physical examples:

insert fig 4.13 here

insert p. 111 here