16.06 Lecture 21
Root Locus Rules

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October 23, 2003

Today's Topics

1. Angle and magnitude conditions

2. Root locus rules

Reading: 6.1, 6.2, 6.3

1 Angle and Magnitude Conditions

Recall what we saw last lecture. The closed-loop C.E. is

$$G_c GH + 1 = 0$$

$$G_c GH = -1$$

$$K \frac{(s+a_1)(s+a_2)\cdots(s+a_m)}{(s+b_1)(s+b_2)\cdots(s+b_n)} = -1$$

The closed-loop poles are the values of s for which the vector $G_c GH$ has a length of one and a phase angle of $\pm(2n+1)180°$. The vector $G_c GH$ can be computed graphically by considering appropriate products of the factors $s + a_i$ and $s + b_k$.

1. **Angle condition**

2. **Magnitude condition**

Note that the A_i may be taken to be 1 if there are no open-loop zeroes.

2 Applying the angle and magnitude conditions

We see that there is a two-stage process:

I (a) Select (guess) a trial point in the s-plane for the locus

(b) Check the angle condition

(c) Probably not right - move point until it is - now you have a closed-loop root

(d) Repeat to develop the complete locus

II Select a desired closed-loop root and calculate the necessary K

3 Rule 1

For $K = 0$, the closed-loop poles coincide with the open-loop poles.

See examples from Lecture 20.

4 Rule 2

For $K \to \infty$, the closed-loop poles approach the open-loop zeroes.

5 Rule 3

There are as many locus branches as there are open-loop poles.

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6 Rule 4

If there are fewer open-loop zeroes than poles, those branches for which there are no open-loop zeros left to go to tend to infinity along asymptotes. The number of asymptotes is equal to the number of open-loop poles minus the number of open-loop zeroes.

$$G_cGH = K\frac{(s+5)}{(s+1)(s+2)}$$

The angle condition tells us:

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7 Rule 5

The directions of the asymptotes are found from the angle condition. The asymptote angles α satisfy

$$\alpha = \frac{\pm(2i+1)180}{n-m}$$

i is any integer, n is the number of poles and m is the number of zeroes.

Consider the previous examples:

8 Rule 6

All asymptotes intersect the real axis at a single point, at a distance ρ_0 to the origin.

$$\rho_0 = \frac{\text{(sum of OL poles) - (sum of OL zeroes)}}{n - m}$$

9 Rule 7

Loci are symmetrical about the real axis, since complex open-loop poles and zeroes occur in conjugate pairs.

We already knew this ...

10 Rule 8

Sections of the real axis to the left of an odd total number of open-loop poles and zeroes on this axis form part of the loci.

We have seen this before ...

11 Rule 9

Points of breakaway from or arrival at the real axis may exist... However, the precise location of these points is not usually of much interest and a rough approximation is okay.

If no other poles and zeroes are close by, the breakaway point will be halfway. A pole tends to push the breakaway point away. A zero tends to attract the breakaway point.

12 Rule 10

Angles of departure from open-loop poles and angles of arrival at open-loop zeroes are important because aero/astro toys can have complex conjugate poles and zeroes near the $j\omega$ axis. We determine these angles by applying the angle condition to a trial point very close to the pole or zero.

Example 1:

Example 2: