MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

PROBLEM SET 2
Due: 9/18/2003

Problem 1: Block Diagram Manipulation

Do Problem 3.27 on page 95 of Van de Vegte. Note that there are two block diagrams here–you need to find the closed-loop transfer function $C(s)/R(s)$ for each one.

Problem 2: Linearization

The following diagram represents a typical spring-mass-dashpot system subject to an external force $f(t)$. The mass of the object is m, and the damping factor of the dashpot is c. However, the force due to the spring is non-linear and is given by $f_k(x) = kx^{2/3}$.

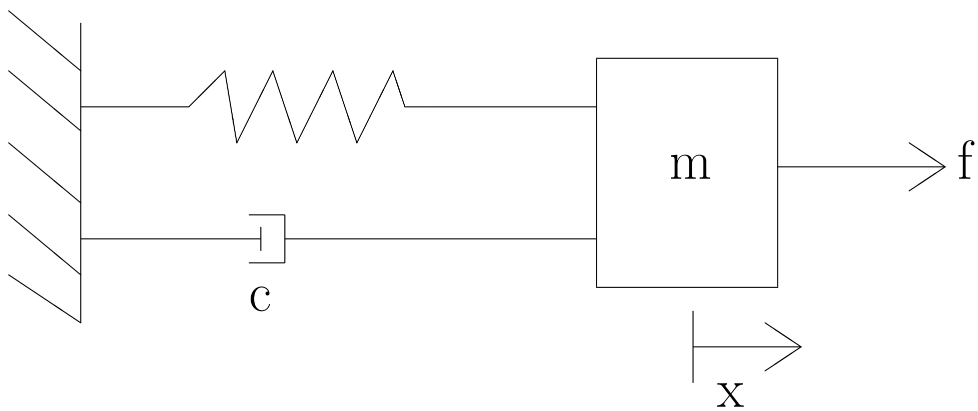

Figure 1: Spring-Mass-Dashpot System

1. Find the differential equation of motion for the system.
2. Linearize the equation about the point $x = x_0$. Make sure you explain what you're doing at each step of the linearization process.
3. Using your linearized model, derive the transfer function of the system. Take the external force to be the input, and the position of the mass to be the output.
4. What is the main limitation of this linearized model? Why does this limitation exist? (Hint: Think back to the Taylor Series expansion. What assumptions are you making when you drop the higher-order terms? One or two sentences of explanation is sufficient.)

Problem 3: Steady State Errors

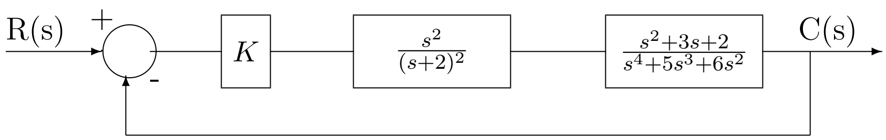

Figure 2: Unity Feedback System

1. For the system in Fig. 2, write the forward path transfer function in the general form given in equation (4.11) on Page 103 of Van de Vegte. What is the type number of this system? What is the gain?

2. What is the steady-state error of this system in response to a unit step input?

3. Without using the Final Value Theorem, explain why the steady-state error of a type 1 system in response to a unit ramp input is constant. (Note: this question is unrelated to the first two parts of this problem.)

Problem 4: Sensitivity to Parameter Variations

The transfer function for a DC motor is given by:

$$G(s) = \frac{\Omega(s)}{E_f(s)} = \frac{K_t/R_fB}{T_ms+1} \tag{1}$$

where Ω is the angular velocity (the "rpm") of the motor shaft, E_f is the applied voltage, K_t is the motor torque constant, R_f is the resistance in the motor circuit, B is the damping constant, and T_m is the motor time constant. (See Van de Vegte, Chapter 2.4 for more details.)

Part 1:

Figure 3 shows the block diagram for the motor in an open loop configuration.

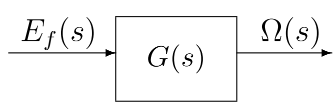

Figure 3: Open Loop

1. Find $\omega_{ss} = \lim_{t\to\infty} \omega(t)$ for the open loop system with a unit step input. (Note on notation: $\Omega(s)$ is the Laplace transform of $\omega(t)$.)

2. Using your formula for ω_{ss} in response to a unit step input, substitute $T_m = 0.1$, $K_t = 1$, and $R_fB = 1$ to get a numerical value of the steady-state output. Then recalculate ω_{ss} with $K_t = 2$. By what percentage did the steady-state output value change?

3. Calculate the static sensitivity (as defined in lecture) to the parameter K_t. Use the nominal values $T_m = 0.1$, $K_t = 1$, and $R_fB = 1$. How do you interpret the value of the static sensitivity?

4. Based on your answers to the last two questions, why would open-loop control be a bad idea if you wanted to accurately control the speed of the motor shaft?

Part 2:

Figure 4 shows the block diagram for the motor in a closed loop configuration. We have now added a proportional controller, K, and a feedback path.

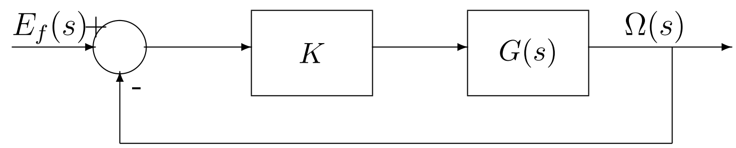

Figure 4: Closed Loop

1. Find $\omega_{ss} = \lim_{t\to\infty} \omega(t)$ for the closed loop system with a unit step input.
2. Using your formula for ω_{ss} in response to a unit step input, substitute $K = 1$, $T_m = 0.1$, $K_t = 1$, and $R_f B = 1$ to get a numerical value of the steady-state output. Then recalculate ω_{ss} with $K_t = 2$. By what percentage did the steady-state output value change?
3. Repeat the previous question for a gain of $K = 10$.
4. Calculate the static sensitivity (as defined in lecture) to the parameter K_t, for controller gains $K = 1$ and $K = 10$. Use the nominal values $T_m = 0.1$, $K_t = 1$, and $R_f B = 1$.
5. Based on your answers to the previous questions, how does closed-loop control compare to open-loop control for the DC motor? How does the value of the gain affect the impact of parameter variations?

Problem 5: Disturbance Rejection

Consider the following system with controller G_c, reference input R, and disturbance input D:

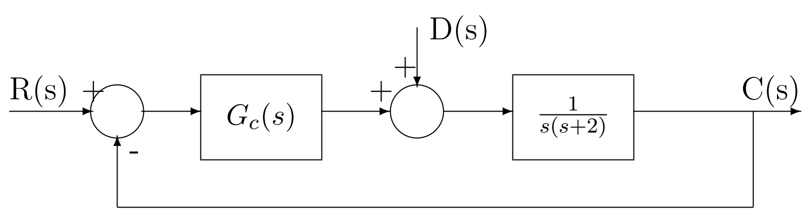

Figure 5: Closed Loop System with Disturbance Input

1. If $G_c(s)$ is a proportional controller, $G_c(s) = K_c$, what value of K_c will cause the steady-state error for a unit ramp reference input to be 0.1?
2. If we want a zero steady-state error for a unit ramp reference input, what form should the controller G_c take?
3. For a unit step disturbance input, what form of controller G_c should we choose to make the steady-state error zero?