MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

PROBLEM SET 3
Due: 9/25/2003

Problem 1: Review of Gain, Type Number, and Steady-State Errors

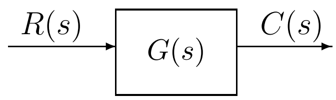


Figure 1: Problem 1

In Figure 1, the transfer function between input and output is given by:

$$G(s) = \frac{10(s+2)(0.25s+1)}{(0.1s+1)(s+5)}$$

1. Convert $G(s)$ to root locus form and find the root locus gain.
2. Convert $G(s)$ to standard form and find the gain, as defined in Van De Vegte, Equation (4.11).
3. What is the type number of the system? If you closed the loop around G(s) with unity feedback, and gave it a step input R, what would the steady-state error be?

Problem 2: Graphical Determination of Residues

For each of the following systems:

- Write down $C(s) = G(s)R(s)$.
- Find the root locus gain of $C(s)$.
- Plot the poles and zeros of $C(s)$.
- Graphically determine the residues for $C(s)$.
- Find the output $c(t) = \mathcal{L}^{-1}[C(s)]$

1. $G(s) = \frac{3}{2}\frac{\frac{s^2}{6}+\frac{5s}{6}+1}{\frac{s^2}{4}+\frac{5s}{4}+1}$ and R is a unit step.
2. $G(s) = \frac{s+1}{s^2-25}$ and R is an impulse (i.e. $R(s) = 1$).
3. $G(s) = \frac{2s+4}{s^2+2s+2}$ and R is a unit step.
4. $G(s) = \frac{4s+8}{(s+1)(s^2+6s+18)}$ and $r(t) = \mathcal{L}^{-1}[R(s)] = e^{-5t}$.

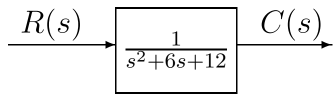


Figure 2: Problem 3.1

Problem 3: Transient Response Characteristics

1. For the system in Figure 2:

   (a) Plot the pole-zero pattern.

   (b) Calculate the damping ratio ζ, the undamped natural frequency ω_n, the damped natural frequency ω_d, and the time constant T.

   (c) Is this system overdamped, underdamped, or critically damped? What is the type number? What is the order?

2. Figure 3 shows the unit step response of a system with a pair of quadratic poles.

   (a) From the data in the graph, estimate ζ, ω_n, ω_d, T, and the gain of the transfer function. Indicate clearly which data points you use to find these quantities. (Your measurements don't have to be super accurate, but they should be fairly close to the true values.)

   (b) What are the poles of the transfer function for this system? Write down the transfer function for this system.

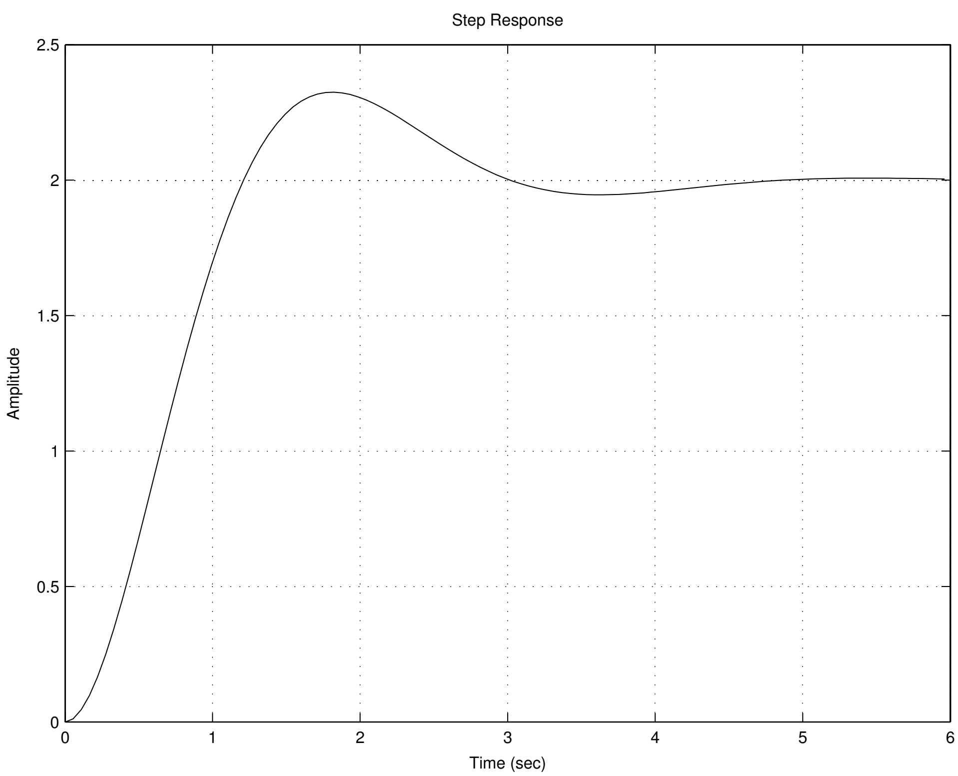
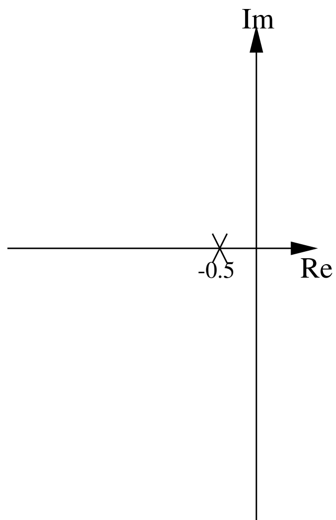


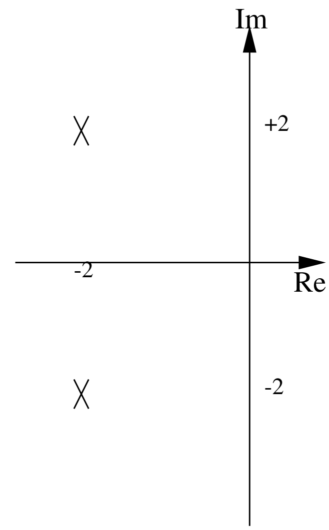
Figure 3: Problem 3.2

Problem 4: Dominant Modes

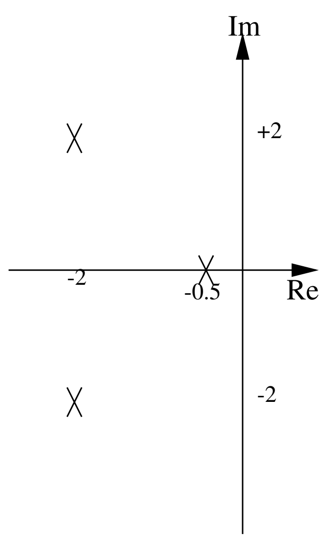
Consider the following pole-zero maps (not drawn to scale):



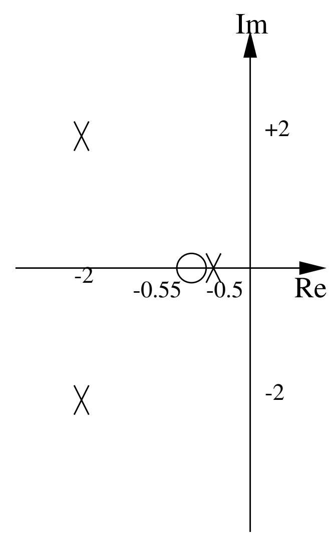
(1)



(2)



(3)



(4)

1. For each system, set the "standard" gain (as defined on p.104 of Van de Vegte) to 1. Then write the transfer function of each system in root locus form.

2. For the system in (1) indicate the time constant. For the system in (2) indicate ζ, ω_n, ω_d and the time constant.

3. What is the step response of each system (find c(t))?

4. Plot the step response of each system on the same plot (you can use the Matlab **step** command).

5. What is the dominant mode in (3)? In (4), which mode dominates initially, and which dominates later in time? (This effect is known as a "long tail", and you will see it when we work with the Quansers.) Why do you think the zero at $s = -0.55$ has this effect?

Problem 5: Effects of Gain on Transient Response

1. Do Problem 4.17 in Van de Vegte.

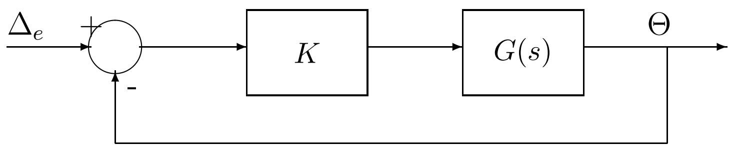
2. In the F-8 aircraft, the longitudinal dynamics (i.e. the pitch angle θ in response to an elevator deflection δ_e) include a "short period" mode described by the following transfer function:

$$G(s) = \frac{1}{s^2 + 2s + 12.9}$$

(For this problem, we will neglect all other dynamic modes.)

(a) Where are the poles of $G(s)$? Is this system underdamped, critically damped, or overdamped?

Now let's close the loop around $G(s)$ and add a proportional controller K:



(b) Our goal is to reduce the pitch angle oscillations. In this case, we will actually choose a negative gain for our proportional controller K (you will understand why later on in the course). If we set $K = -10$, where are the closed-loop poles of the system? Is the system underdamped, critically damped, or overdamped?

(c) Repeat part (b) for a gain of $K = -12$.

(d) Repeat part (c) for a gain of $K = -20$. What is different about the closed-loop system now?