16.06 lab report: example Results & Conclusions sections. Note that figures and tables are all labeled properly, and are referred to in the text. The conclusions are not long, but clearly state the major points of the lab.

Results

Due to small variations in system parameters, each Quanser is unique. Because of this uniqueness it is important to note which system was characterized in the first lab, so that information can be applied in the succeeding labs. We were using Quanser # 3. The counterweight was at the farthest extended position.

By trial and error we found the input voltage that holds the Quanser in the horizontal position. We manually adjusted the voltage while eyeing the position of the Quanser. Our Quanser maintains a horizontal position with an input of 8.6 volts. The zero of elevation angle corresponds to the Quanser sitting on the table. We called horizontal 19.9°. Our system was relatively well behaved, after the initial transients decayed it stayed within +/- 0.1° of the steady state value.

Part 1: DC Gain

Next we found the steady state response of the system to five evenly spaced input voltages. The minimum voltage corresponded to the Quanser barely lifting off the table. The maximum voltage was the signal that caused the Quanser to settle to an elevation angle just below the hard stop at the upper side of the range of motion. Refer to Table 1 below for these results. Data was sampled for 90 seconds after the initial transient decayed. The steady state elevation angle is taken as the average value over the sample period.

Input Voltage [V]	Mean Elevation Angle [°]
6.5	1.6
7.4	8.7
8.3	19.2
9.2	31.0
10.1	52.4

Table 1. Steady state elevation angle as a function of input voltage, evenly distributed by voltage over the full range of motion.

We would like to model the Quanser as a linear system. In reality it is a highly nonlinear system. To get a handle on how to approximate the system as linear at various operating points, we could view the data in Table 1 in a more useful way. We can approximate the derivative of the output with respect to the input, evaluated at each operating point by using a finite difference approximation for the derivative. We will use the backward difference method at each point, with (5.6V, 0°) as the zero point to approximate the linear behavior of the system. See Table 2 below for a summary. For a DC gain value that is representative of the full range of motion the median value, 11.7, was selected.

Operating Point [V]	DC Gain [°/V]
6.5	1.8
7.4	7.9
8.3	11.7
9.2	13.1
10.1	23.8

Table 2. Approximate linear behavior of the system at five operating points.

It is evident in Table 2 that the system becomes more sensitive to changes in input as the elevation increases. If we wanted to approximate the behavior of the system as linear, we need to be aware that this approximation is only reasonable near the selected operating point.

Part 2: Step Response and Transfer Function Model

In order to create a second order linear model of the Quanser, we measured the step response of the system at three operating points. The first step response was adjusted so that the output would start at a small non zero value and peak at a value just smaller than the maximum. Unfortunately we did not maximize the amount of useful data in our 90 second buffer for the first step response, and therefore did not catch the settling time or the steady state value. A reasonable estimate of the steady state value was obtained by graphically selecting the center of oscillations for the initial transients. The steady state value for the first step response was 32°. The damping ratio was found using the percent overshoot. The percent overshoot for this step response was 81%, which yielded a damping ratio of 0.067. Since the settling time was not present in the sample, the peak time was used instead. The peak time was found to be 7 seconds, which yielded a natural frequency of 0.45 radians/second. Using these values and the median dc gain of 11.7, the second order linear model is complete.

Using this same procedure and the data from the first step response in part 8, a damping ratio of 0.044 and a natural frequency of 0.47 radians/second were found. This step response is representative of the system behavior at low angles of elevation. These results for damping and natural frequency are reasonably close to those found in part 7. For the second step response measured in part 8, the damping ratio was found to be 0.36, and the natural frequency was 0.35 radians/second. This step response is a measure of the system behavior at high angles of elevation. These values for damping and natural frequency are drastically different than those obtained in part 7. This is an indication that our second order linear model will be a poor representation of the behavior at high angles of elevation.

Using simulink the second order linear model was simulated with the damping ratio and natural frequency from part 7, and the input from the two step responses in part 8. In order to make a meaningful comparison the experimental data was shifted in time so the start of the step response corresponded to a time of zero. It was also necessary to shift the model data in elevation, so that the elevation at time zero for the model was equal to the elevation at time zero for the experiment. Refer to Figure 1 for a summary of these data.

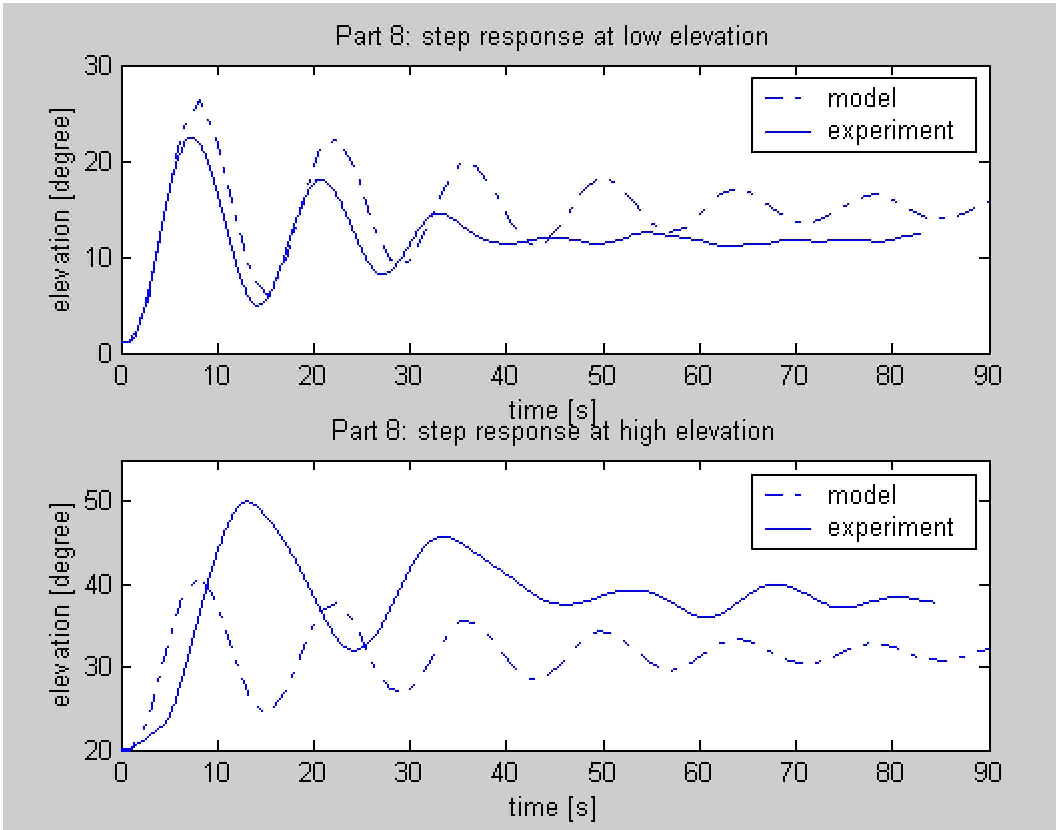

Figure 1. Comparison of second order linear model with dynamic response of Quanser at low elevations and high elevations. From these plots, it is evident that G_{dc} is too high for low elevations and too low for high elevations.

A major discrepancy between the model and experiments is that the steady state value is predicted to be to large at low elevations and too small at high elevations. This is due entirely to the DC gain. The gain for the model is too large at low elevations and too small at high elevations. It is not possible to simultaneously improve the models accuracy at low elevations and high elevations. The DC gain was selected as the median value from the experiments, so that it would be as reasonable as possible over the full range of motion. For this reason I have chosen not to change the values for my model.

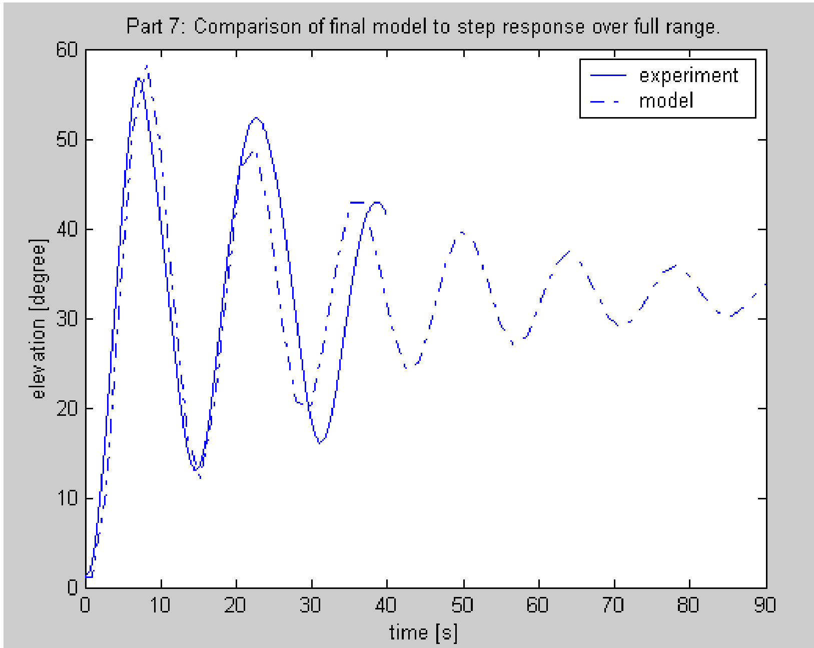

Figure 2. Comparison between real system and second order linear model to a step response of magnitude 2.7 V. The model parameters are $\zeta = 0.067$, $\omega_n = 0.45$ rad/s, $G_{dc} = 11.7$

The model is very accurate for the first 20 seconds of the step response. It is evident that the natural frequency is slightly too high for the model. I will modify this value to 0.43 rad/s and repeat the simulation.

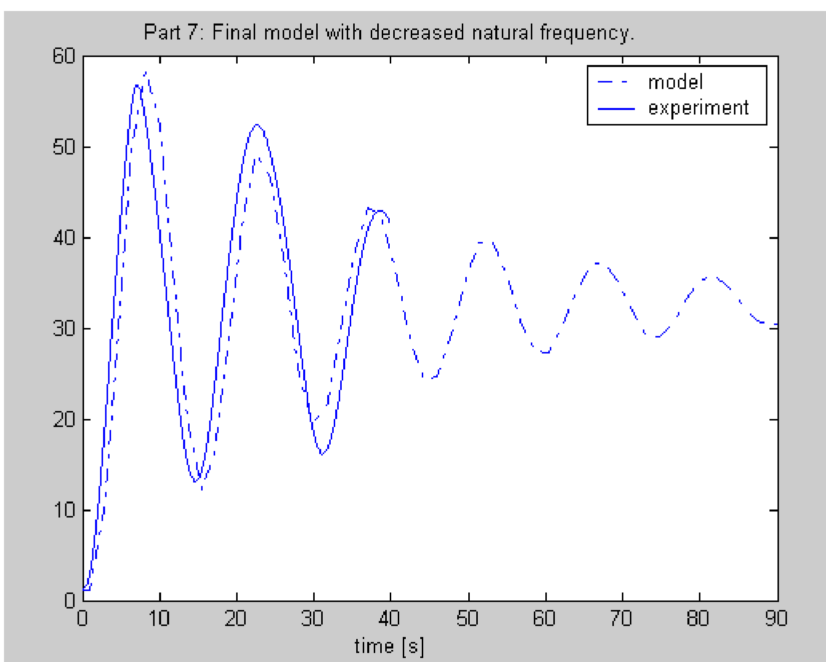

Figure 3. Comparison with revised model. This is the best agreement I could obtain. The model parameters are $\zeta = 0.067$, $\omega_n = 0.43$ rad/s, $G_{dc} = 11.7$.

Discussion of Results / Sources of Error

The second order linear model developed represents the Quanser reasonably well for elevations near the horizontal position. As the elevation reaches the extremes of the range of motion, the model becomes less accurate. This behavior is due to the non-linearity in the DC gain. Physically this non linearity is due to the ground effect of the table on the performance of the thrusters and also to a break down in the small angle approximation at high angles far from the horizontal position.

Maximizing the accuracy of the model at the horizontal position is the most robust way of modeling the system. If we were to decrease the error at low angles it would increase the error at high angles, and vice versa. When we eventually implement this model in a controller, we would expect the system to be well behaved near the horizontal position, and ill behaved at the extremes.

There are several contributing factors that limit the accuracy of the model. Not all the physics are included in the model. The major sources of error in the model are:

1. Aerodynamic disturbances due to the flow generated by other Quansers, air conditioning, people, etc.
2. Ground effects on the performance of the thrusters.
3. Inaccuracies of the small angle approximation near the extremes in the range of motion.
4. Inertia and drag of the connecting rods and wires.
5. Friction in the pivot.
6. Noise in the sensors.
7. Interference from the closed loop travel control on the elevation axis. In reality the system has three degrees of freedom, not one.

Conclusion

The objective of this lab was to model the elevation dynamics of the Quanser. The hypothesis was that the dynamics from input voltage to output angle can be modeled as a second-order, linear, underdamped system. This hypothesis was tested by measuring several different step responses and then attempting to fit a second-order model to the data.

Qualitatively, a very lightly damped second-order model appears to be a good representation of the physical system. The best model parameters were found to be damping ratio of 0.067, natural frequency of 0.43 rad/s and a DC gain of 11.7°/V. However, an exact match between the model and experimental data could not be obtained. This discrepancy is attributed to the nonlinearity of the Quanser dynamics as well as disturbance sources such as air currents.