16.06 Lecture 5
Steady-State Errors

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Today's Topics

1. Steady-state errors

2. The importance of integrators

3. PI control

Reading: 4.3, l.n.

1 Introduction

Recall from the last lecture that high loop gains reduce the sensitivity to parameter variations and disturbance inputs.

Today, we will see that they also reduce steady-state errors in feedback systems.

2 Steady-State Errors

Consider the unity feedback system:

How do we determine the steady-state error, e_{ss}?

Use the final value theorem:

$$\begin{aligned} e_{ss} &= \\ &= \end{aligned} \quad (1)$$

For $G(s)$ the following general form is assumed:

$$G(s) = \quad (2)$$

3 Definitions

(a) **Gain**

(b) **Type number**

(c) **Position error constant**, K_p

(d) **Velocity error constant**, K_v

(e) **Acceleration error constant**, K_a

Equation (2) shows that

So, equation (1) can be written:

$$e_{ss} = \tag{3}$$

4 Table of steady-state errors

Using equation (3), we can construct a handy table:

e.g. type 2 system, unit ramp input

5 A physical explanation of the table

5.1 Consider a type 0 system

Type 0, step input:

If c is constant, then e_{ss} must be

This means:

Type 0, ramp input:

5.2 Consider a type 1 system

Type 1, step input:

- from table, $e_{ss} = 0$
- remember for an integrator:
- if c levels off to a constant value, then
- notice the gap in time for $e(t)$ and $c(t)$. This accounts for

6 PI control

This section includes some extra reading for you. This should give you some hints as to how the concepts we have seen so far are important for controller design. We will revisit this example in more detail in later lectures.

So far we have seen that integrators seem like good things to put in our controller...

(a) Suppose you have a plant $G(s) = \frac{K}{s(\tau s+1)}$ and you add an integrator to make it type 2.

First draw the block diagram:

The closed-loop transfer function will be:

$$\frac{C}{R} = \frac{\frac{K}{s^2(\tau s+1)}}{1 + \frac{K}{s^2(\tau s+1)}} = \frac{K}{\tau s^3 + s^2 + K}$$

and the characteristic equation of the system will be

$$\tau s^3 + s^2 + K = 0$$

This is bad! The system has roots in the right half plane. We will see more about the importance of the characteristic equation in the next lecture.

(b) Is there a way that we can increase the type number of this system without compromising stability? One common form of controller is a PI controller: proportional plus integral control, $G_c = K_p + \frac{K_i}{s}$.

For the plant in this example, the new CE is

$$\tau s^3 + s^2 + KK_p s + KK_i = 0$$

As a designer, you get to pick the values of K_p and K_i to make the system behave in the way you want (i.e. by changing the system characteristic equation). We will see more about designing controllers in Lectures 22-25.