16.06 Lecture 9
Transient Performance and the Effect of Zeroes

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Today's Topics

1. Transient response performance measures

2. The effect of zeroes

Reading: 5.1, 5.2

1 Introduction

We saw last lecture that the response of many systems is dominated by a particular mode. Often for aero/astro systems, the response is dominated by a complex conjugate pole pair. Using the dominant pole concept simplifies our design problem, because we can focus on locating this dominant pair in a satisfactory position. Note that this is the approach that we are taking for the Quansers!

2 Transient response performance measures

(a) Assume that at first, the response is dominated by a complex pair close to the $j\omega$ axis.

insert Fig. 5.1

(b) Referring to Fig 5.1, consider the following definitions:

(i) Settling time, T_s, is the time required for the response to come permanently within a 2% band around the steady-state value.

$$T_s =$$

(ii) P.O. is the maximum percentage overshoot over the steady-state response.

$$P.O. =$$

(iii) Peak time, T_p, is the time to the maximum peak of the response.

$$T_p =$$

(iv) Rise time, T_r, is the time at which the response first reaches the steady-state level.

$$T_r =$$

(c) Performance guide times

(i) P.O. depends only on ζ. Recall that $\zeta = \cos\phi$, therefore to avoid excessive overshoot, ϕ may not be close to $90°$.

[ζ is usually $0.4 - 0.7 (25\% - 5\%)$]

(ii) T_s - to decrease:

(iii) T_p and T_r - to decrease:

3 Effects of zeroes

We have talked a lot about system poles and the correlations between their positions in the s-plane and the response of the system. But what about the system zeroes? What effect do they have on the response? This question is especially important for design, because often when we design a controller, we will use a zero to improve the system dynamics. You will do this yourself in the second lab.

3.1 Where do zeroes come from?

(a) May be part of the plant (space shuttle, satellites, X-29A – come from the dynamics)

Example: Attitude control of a rocket

(i) Take moments about the CM. Assume moment of inertia of engine is zero and θ and δ are small:

(ii) Use $F_n = C_n\theta$ for the aerodynamic force and linearize the equation above:

(iii) Compute the transfer function between δ and θ:

(We must stabilize by closing the loop around θ.)

Now include the inertia of the rocket engine:

(i)-(ii) Take moments about the CM and linearize:

(iii) Compute the transfer function between δ and θ:

where $b^2 = l_2 T / I_E$ and $a^2 = l_1 C_n / (I + I_E)$.

(b) May be part of the compensator (usually are).

e.g. PI control:

$$G_c(s) =$$

(c) May come from the feedback path

3.2 Relationship between open-loop poles and zeroes and closed-loop poles and zeroes

$$T(s) = \frac{C(s)}{R(s)} =$$

- Zeroes of $T(s)$ are
- Poles of $T(s)$ are

Example: