Massachusetts Institute of Technology

# 16.07 Dynamics

# Problem Set 2

Out date: Sep 15, 2004

Due date: Sep 22, 2004

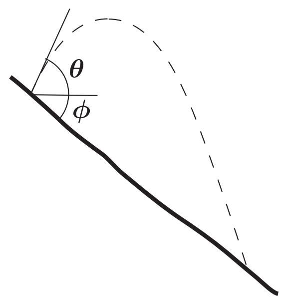
| | Time Spent [minutes] |
|---|---|
| Problem 1 | |
| Problem 2 | |
| Problem 3 | |
| Problem 4 | |
| Study Time | |

*Turn in* **each problem** *on* **separate sheets** *so that grading can be done in parallel*

## Problem 1

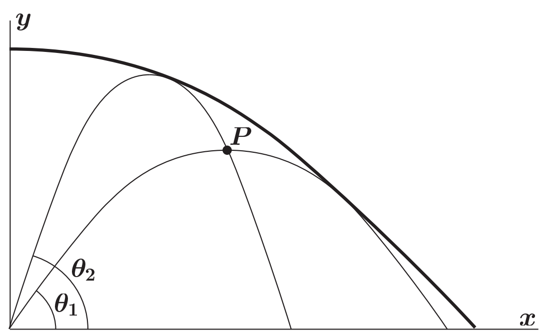
### Part A

A boy stands at the peak of a hill which slopes downward uniformly at angle $\phi$.




- Assuming that he always throws at the same speed $v_0$, regardless of the angle, find the angle $\theta$ from the horizontal (as a function of $\phi$), he should throw a rock so that it has greatest range?

Assume now, that $\phi = 0$. We want to calculate the equation of the envelope of the parabolic trajectories of a rock thrown at the same speed $v_0$, but with different angles.




We will do this is two stages:

- Find the angle at which he should throw the rock so that the trajectory goes through a given point $P = (x, y)$.

*Hint: you should obtain a quadratic equation for* $m = \tan\theta$ *(the following identity may prove useful* $1+\tan^2\theta = 1/\cos^2\theta$*). Depending of the roots of the equation, the problem will have two solutions, one solution, or no solution.* Note that the envelope is made up of all the points for which there is only one solution. In particular, for all the points inside the envelope we have two solutions, and for all the points outside the envelope we have no solutions.

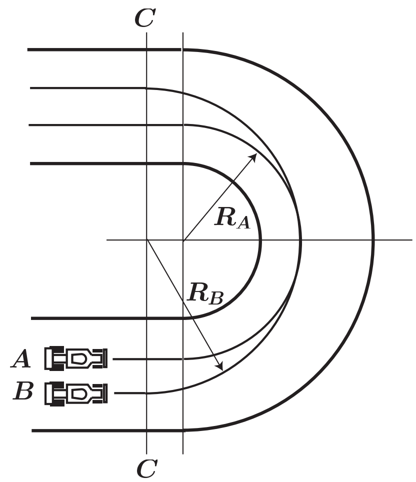
- Show that the equation of the envelope is

$$y = \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}\ .$$

(*Neglect air resistance for all the questions above.*)

**Part B**

The figure below shows two racing cars and the paths they follow to negotiate the curve on the unbanked track ($R_A = 50$ m, and $R_B = 58$ m). If each car has a constant speed limited to that corresponding to a lateral (normal) acceleration of $0.9g$, determine the times $t_A$ and $t_B$ for both cars to complete the turn as delimited by the line $C - C$.

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**Problem 2**

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**Part A** (Merriam)

To simulate a condition of "weightlessness" in its cabin, an aircraft travelling at 500 km/h moves on a sustained vertical curve as shown. At what rate $\dot{\beta}$ in degrees per second should the pilot drop his longitudinal line of sight to effect the desired condition?

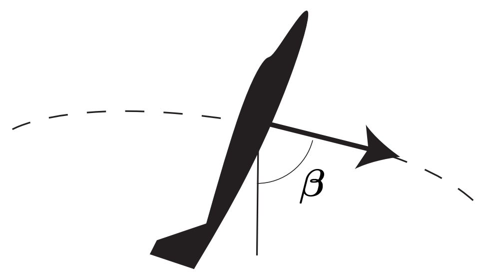

**Part B**

During the reentry of a spacecraft into the earth's atmosphere, the local acceleration of the spacecraft can be represented by the sum of two components: gravity and that due to air resistance. The gravitational acceleration at this height is $g = 9.5$ m/s$^2$, and the air resistance acceleration is 15 m/s$^2$ and is directed opposite to the velocity. The velocity of the spacecraft has a magnitude of 4000 m/s, and forms an angle of $\beta = 78°$ with the vertical. Determine:

a) the rate $\dot{v}$, which the speed is changing,

b) the radius of curvature of the trajectory, and,

c) the rate $\dot{\beta}$ at which the direction of the velocity vector is changing.

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**Problem 3**

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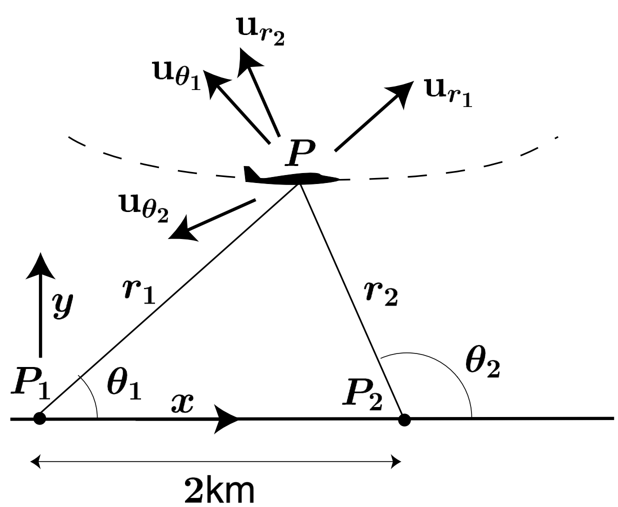
**Part A**

The flight of an aircraft is being tracked from two stations located at the fixed points $P_1$ and $P_2$ which are situated 2 km apart. At each station we can measure the elevation angle and the first and second time derivatives of the elevation angle.



At a given instant, they measure the following values

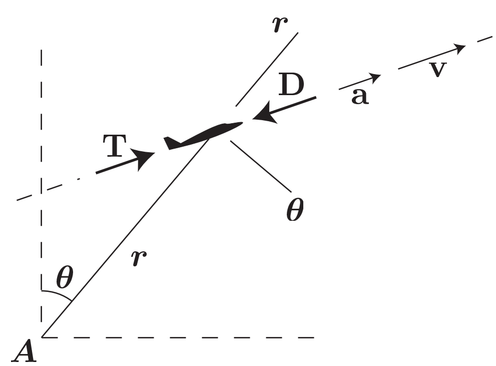
$$\begin{aligned}\theta_1 &= 25^\circ \\ \dot{\theta}_1 &= -0.057 \text{ rad/s} \\ \ddot{\theta}_1 &= 0.017 \text{ rad/s}^2 \\ \theta_2 &= 140^\circ \\ \dot{\theta}_2 &= -0.114 \text{ rad/s} \\ \ddot{\theta}_2 &= -0.0326 \text{ rad/s}^2.\end{aligned}$$

From these values you are asked to determine:

a) the position, velocity and acceleration vectors of the aircraft referred to the $x-y$ axis, and

b) the instantaneous radius of curvature of the aircraft trajectory.

**Part B** (Meriam/Kraige)
A 5.0 Mg aircraft is tracked by radar at $A$ below its straight line of flight. At the instant shown, radar gives $r = 20.0$ km, $\dot{r} = 88.0$ m/s, $\theta = 30^o$, $\dot{\theta} = 6.14 \times 10^{-3}$ rad/s, and $\ddot{\theta} = 5.18 \times 10^{-5}$ rad/s$^2$. If the jet engine thrust is $T = 34.0kN$, determine the aerodynamic drag $D$ and the lift $L$ on the aircraft. Also find $\ddot{r}$.

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**Problem 4**

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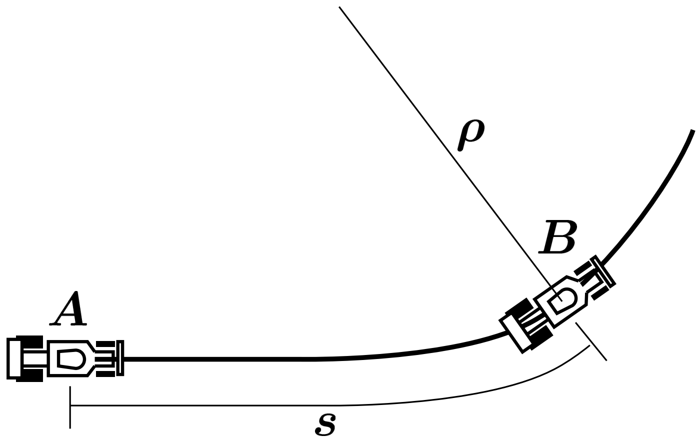
**Part A**
A race car going at 200 Km/h at A decelerates at a constant rate to 80 Km/h at B in a distance $s = 180$ m in order to negotiate an unbanked turn. If the (static and kinetic) coefficient of friction between tires and road is 0.8, and if the car begins to skid at B, determine the radius of curvature $\rho$ of the path at B.




**Part B** (Meriam/Kraige)

A small vehicle enters the top $A$ of a circular path with a horizontal velocity $v_0$ and gathers speed as it moves down the path. Show that the angle $\beta$ which locates the point where the vehicle leaves the path and becomes a projectile, is

$$\beta = \cos^{-1}\left(\frac{2}{3} + \frac{v_0^2}{3gR}\right) .$$

Evaluate your expressions for $v_0 = 0$. Neglect friction and treat the vehicle as a particle.