NAME : ....

Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 9

Out date: Nov 10, 2004

Due date: (Friday) Nov 19, 2004

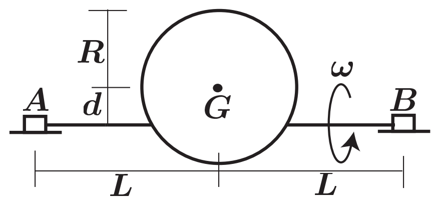
	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

Problem 1

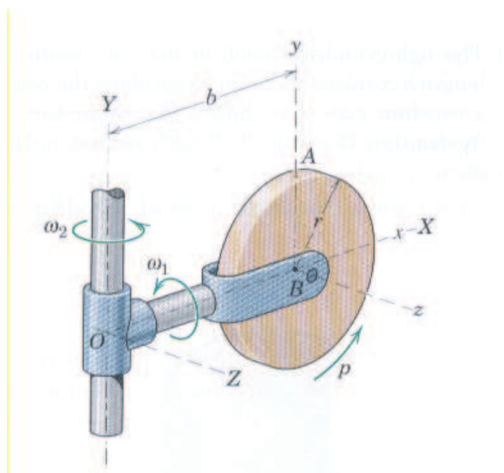
Part A

The 10 kg disc is mounted off center on a shaft that is supported by bearings A and B. If the shaft rotates at a constant angular velocity of $\omega = 10$ rev/s determine the reactions at the bearings when the disc is in the position shown. The mass of the shaft can be considered negligible, and the dimensions are $R = 20$ cm, $L = 30$ cm, and $d = 10$ cm.



Part B (Meriam/Kraige)

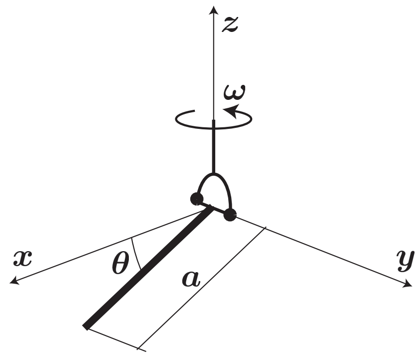
Determine the angular momentum about $\boldsymbol{H}_O$ with respect to O, and the kinetic energy of the disc for the instant represented, when the $x - y$ plane coincides with the $X - Y$ plane. The mass of the disc is m.



Problem 2

Part A

A uniform slender bar of mass ρ per unit length is freely pivoted about the y-axis at the clevis which rotates about the fixed vertical z-axis with a constant angular velocity $\boldsymbol{\omega}$. Gravity acceleration is in the direction of the negative z-axis.



- Calculate the tensor of inertia of the bar with respect to the rotating axes xyz.

- Calculate the steady-state angle θ_0 assumed by the bar.

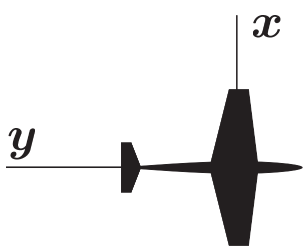
- Calculate the force and the moment exerted by the vertical axel on the bar.

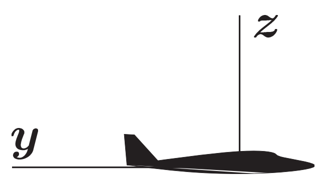
- If the bar is perturbed by a small $\delta\theta$ from the steady state position, determine the period of oscillation of the bar about the position θ_0

Part B

Let xyz be the principal axes through the center of gravity of an airplane. Show that in order for the airplane to make a turn of radius R about a vertical line, with constant speed v, it must bank at an angle $\theta = \tan^{-1}(v^2/Rg)$ and supply a rolling moment in the same direction as θ, equal to

$$\frac{1}{2}(I_z - I_x)\frac{v^2}{R^2}\sin 2\theta \ .$$





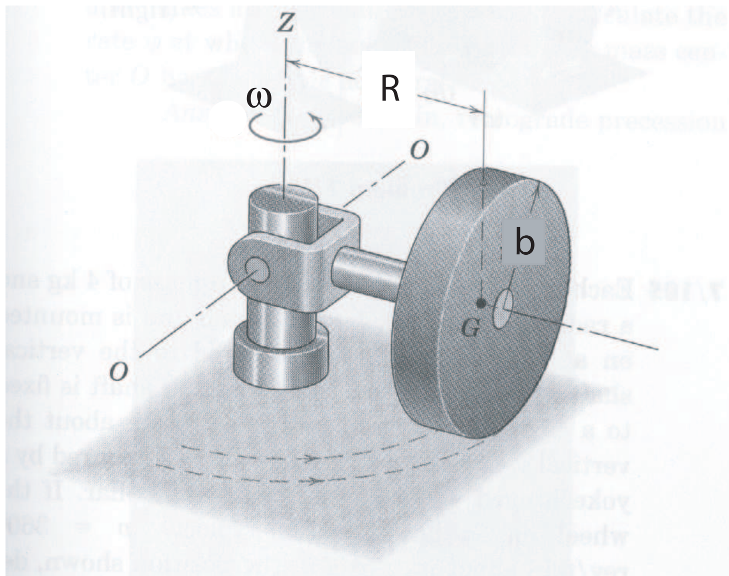
Problem 3 (Klepner/Kolenkow)

Part A

In an old-fashioned rolling mill, grain is ground by a disc-shaped millstone which rolls in a circle on a flat surface driven by a vertical shaft. Because of the stone's angular momentum the contact force with the surface can be considerably greater than the weight of the wheel.

Assume that the millstone is a uniform disc of mass M, radius b, and width w, and that it rolls without slipping in a circle of radius R with angular velocity ω. Find the contact force. Assume that the millstone is closely fitted to the axel so that it can not tip, and that $w \ll R$. Neglect friction.

Hint : If $\Omega^2 b = 2g$, *the force is twice the weight.*



Part B

When an automobile rounds a curve at a high speed, the loading (weight distribution) on the wheels is markedly changed. For sufficiently high speeds the loading on the inside wheels goes to zero, at which point the car starts to roll over. This tendency can be avoided by mounting a large spinning flywheel on the car.

- In what direction should the flywheel be mounted, and what should be the sense of rotation, to help equalize the loading? (Be sure that your methods works for the car turning in either direction)

- Show that for a disc-shaped flywheel of mass m and radius R, the requirement for equal loading is that the angular velocity of the flywheel, ω, is related to the velocity of the car v by,

$$\omega = 2v\frac{ML}{mR^2} \ ,$$

  where M is the total mass of the car and flywheel, and L is the height of the center of mass of the car (including the flywheel) above the road. Assume that the road is unbanked.