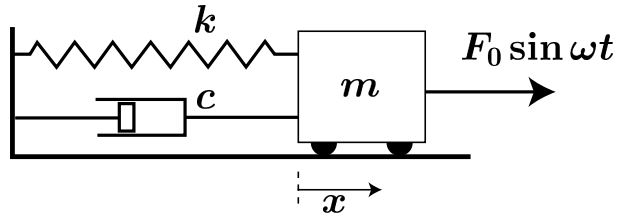


Lecture D33 : Forced Vibration



Spring Force $F_s = -kx,$ $k > 0$

Dashpot $F_d = -c\dot{x},$ $c > 0$

Forcing $F_{ext} = F_0 \sin \omega t$

Newton's Second Law ($m\ddot{x} = \sum F$)

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$

$$\omega_n = \sqrt{k/m}, \quad \zeta = c/(2m\omega_n),$$

Equation of motion

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \frac{F_0}{m} \sin \omega t$$

Undamped Forced Vibration

$$\ddot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \omega t \quad (1)$$

General Solution

$$x(t) = x_c(t) + x_p(t)$$

- $x_c(t)$ is *general* solution of

$$\ddot{x} + \omega_n^2 x = 0$$

... have already seen

- $x_p(t)$ is *any* solution of (1)

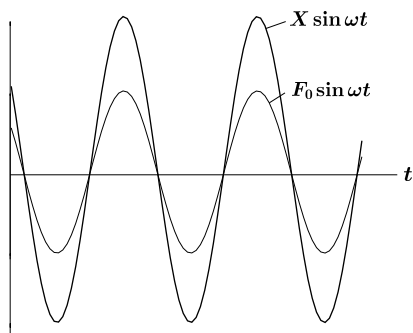
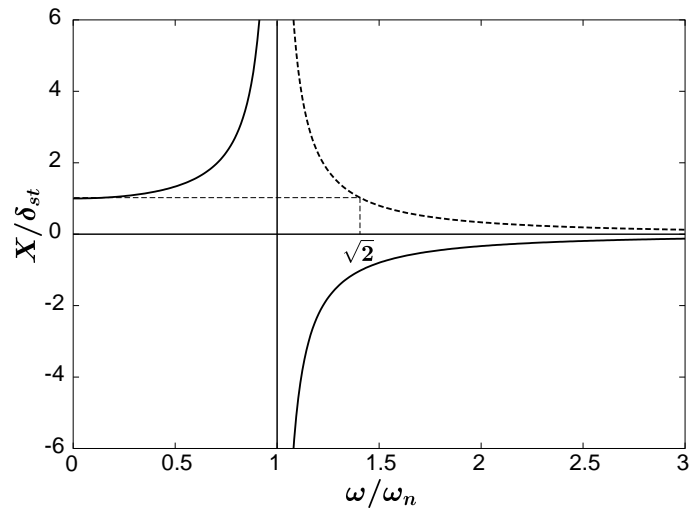
Try $x_p(t) = X \sin \omega t \Rightarrow$

$$X = \frac{F_0/k}{1 - (\omega/\omega_n)^2} = \frac{\delta_{st}}{1 - (\omega/\omega_n)^2}$$

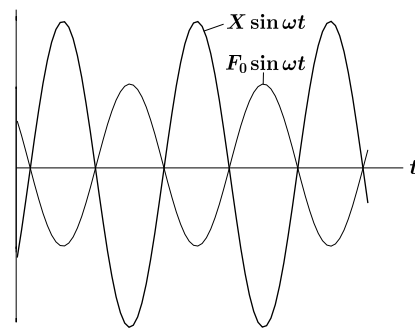
Undamped Forced Vibration

Particular Solution

$$x_p(t) = X \sin \omega t = \frac{\delta_{st}}{1 - (\omega/\omega_n)^2} \sin \omega t$$



$$\omega < \omega_n$$



$$\omega > \omega_n$$

Damped Forced Vibration

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \frac{F_0}{m}\sin\omega t \quad (2)$$

General Solution

$$x(t) = x_c(t) + x_p(t)$$

- $x_c(t)$ is *general* solution of

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0$$

... have already seen

- $x_p(t)$ is *any* solution of (2)

Try

$$x_p(t) = X_1 \cos\omega t + X_2 \sin\omega t$$

or,

$$x_p(t) = X \sin(\omega t - \psi)$$

Damped Forced Vibration

- $x_c(t)$ is *general* solution of

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0$$

... have already seen

- $x_p(t)$ is *any* solution of (2)

Try

$$x_p(t) = X_1 \cos \omega t + X_2 \sin \omega t$$

or,

$$x_p(t) = X \sin(\omega t - \psi)$$

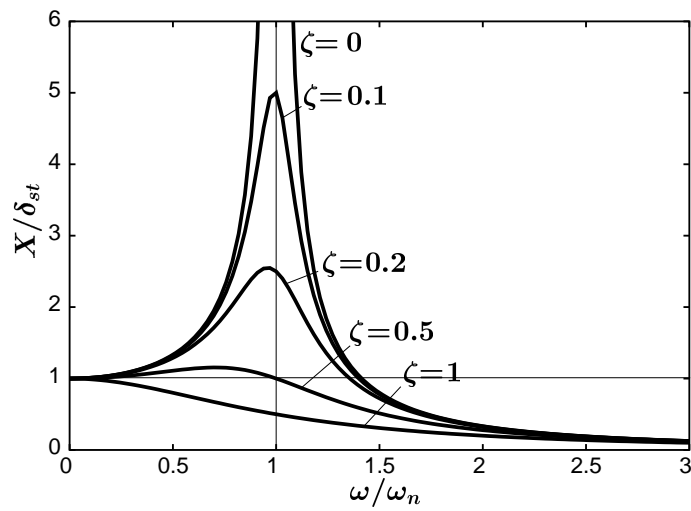
\Rightarrow ...

$$X = \frac{F_0/k}{\{[1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2\}^{1/2}}$$

$$\psi = \tan^{-1} \left[\frac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2} \right]$$

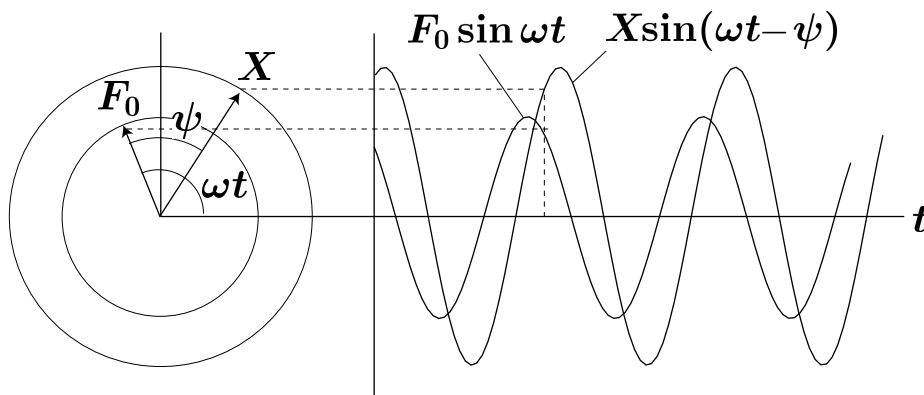
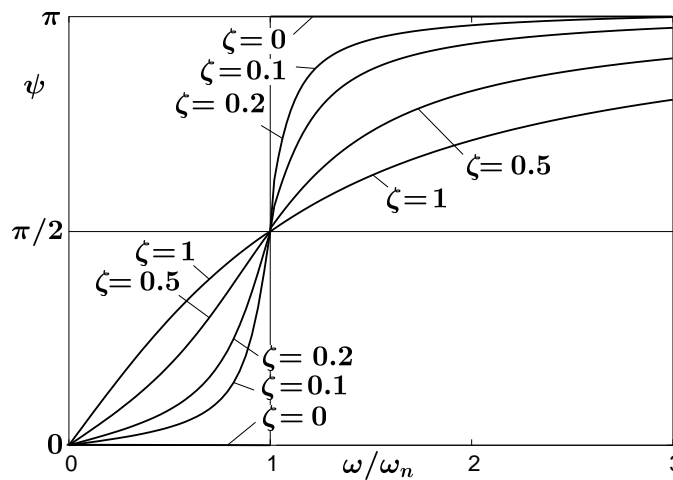
Damped Forced Vibration

$$X = \frac{\delta_{st}}{\{[1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2\}^{1/2}}$$

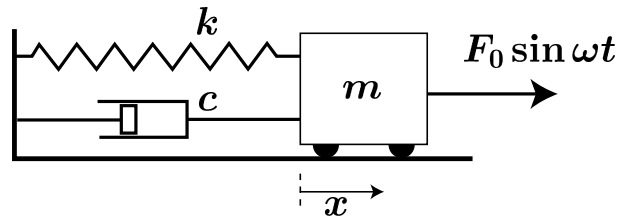


Damped Forced Vibration

$$\psi = \tan^{-1} \left[\frac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2} \right]$$



Vibration Isolation



How much of the applied force is transmitted to the wall?

$$\text{Transmissibility} = \frac{|\text{Transmitted force}|}{|\text{Applied Force}|}$$

Applied Force

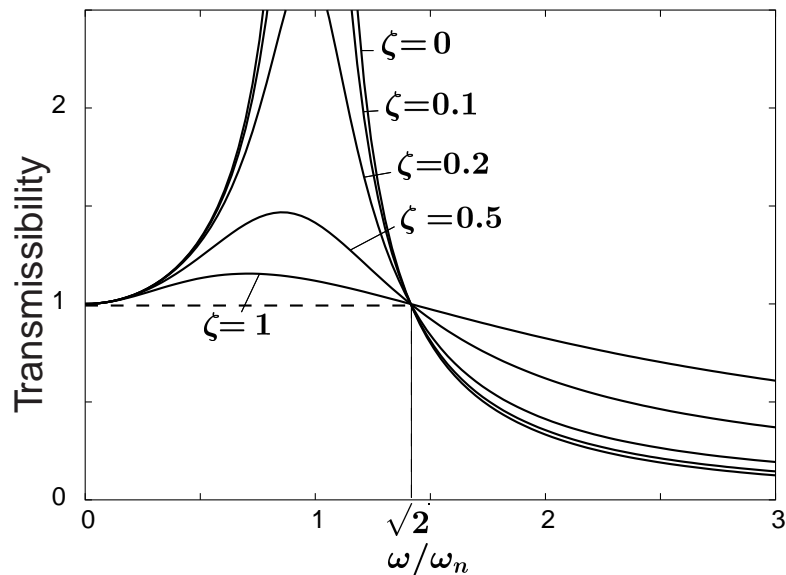
$$F_{ext} = F_0 \sin \omega t$$

Transmitted Force (spring + dashpot)

$$\begin{aligned} F_w &= kx + c\dot{x} \\ &= kX \sin(\omega t - \psi) + c\omega X \cos(\omega t - \psi) \\ &= X\sqrt{k^2 + (c\omega)^2} \sin(\omega t - \sigma) \end{aligned}$$

Vibration Isolation

$$\text{Transmissibility} = \frac{X \sqrt{k^2 + (c\omega)^2}}{F_0}$$
$$= \sqrt{\frac{1 + (2\zeta\omega/\omega_n)^2}{(1 - (\omega/\omega_n)^2)^2 + (2\zeta\omega/\omega_n)^2}}$$



For $\omega/\omega_n > \sqrt{2}$ *damping* increases transmissibility !!

For $\omega/\omega_n < \sqrt{2}$ *having a spring* increases transmissibility !!