

Lecture D1 - Introduction

Introduction

In this course we will study Classical Mechanics. Particle motion in Classical Mechanics is governed by Newton's laws and is sometimes referred to as Newtonian Mechanics. These laws are empirical in that they combine observations from nature and some intuitive concepts. Newton's laws of motion are not self evident. For instance, in Aristotelian mechanics before Newton, a force was thought to be required in order to maintain motion. A lot of the foundations for Newtonian mechanics were laid by Galileo at the end of the 16th century. Newton, in the middle of the 17th century stated the laws of motion in the form we know and use them today, and shortly after, he formulated the law of universal attraction. This led to a complete theory with which he was able to explain many observed phenomena, and in particular the motion of the planets. Nevertheless, these laws still left many unanswered questions at that time, and it was not until later years that the principles of classical mechanics were deeply studied and rationalized. In the eighteenth century, there were many contributions in this direction, such as the principle of virtual work by Bernouilli, D'Alembert's principle and the theory of rigid body dynamics developed by Euler. In the nineteenth century, Lagrange and later Poisson, Hamilton and Jacobi developed the so called analytical or rational mechanics and gave to the theory of Newtonian mechanics a much richer mathematical structure.

Classical Mechanics has its limitations and breaks down where more modern theories such as relativity and quantum mechanics, developed in the twentieth century, are successful. Newtonian mechanics breaks down for systems moving at speeds comparable with the speed of light, and also fails for systems of dimensions comparable to the size of the atom. Nevertheless, for practical engineering applications Newtonian mechanics provides a very good model to represent reality, and, in fact, it is hard to find examples in our field where Newtonian mechanics is not adequate. The most notable perhaps are the relativistic corrections that need to be made for modelling satellite communications.

In this lecture we will introduce Newton's laws of motion and the law of Universal Attraction. Before doing so however, we will define some of the terms which appear in these laws.

Particles, Rigid Bodies and "Real Bodies"

In this course real bodies will be idealized either as particles or as rigid bodies.

A *particle* is a body of negligible dimensions. When the dimensions of the body are unimportant to the description of its motion, we will idealize the body as a particle. For instance, when we consider planetary motion, we will often consider the planets as particles.

A *rigid body* is a body that has a finite size but it does not deform. This will be a useful approximation when the deformation of a body is negligible compared to the overall motion. For instance, we may consider an aircraft as being a rigid body when considering the behavior of the aircraft along its flight path, even though under some specific conditions the deflection of the wing tips may be considerable. In describing the motion of a rigid body, we need to be concerned with its position but also with its orientation.

On the other hand, real bodies have a finite size and are always deformable under loading. In some situations it will be required to consider the deformation of the body when considering its dynamic behavior, but this is outside the scope of this course, except perhaps when we introduce the topics of vibration and waves.

Scalars and Vectors

Scalars and vectors are mathematical abstractions that are very useful to describe many of the concepts in dynamics. You should already have an intuitive idea of what they are. A *scalar* is a single number which is useful to describe the reading of a physical property on a scale. For instance, temperature, length or speed are scalar quantities. On the other hand, *vectors* are much richer entities. They exist in a multi-dimensional space and they have both *direction* and *magnitude*. Velocities, forces and electric fields are examples of vector quantities.

Newton's Laws

Newton's three laws of motion are:

- 1.- *A particle in isolation moves with constant velocity.*

A particle in isolation means that the particle does not interact with any other particle. Constant velocity means that the particle moves along a straight line with constant speed. In particular, it can be at rest. It turns out that the motion (e.g. velocity and acceleration) we observe depends on the reference frame we use. Therefore, the above law can not be verified in all reference frames. The reference frames for which this law is satisfied are called *inertial reference frames*. In some sense, we can say that Newton's first law postulates that inertial reference frames exist.

- 2.- *The acceleration of a particle relative to an inertial reference frame is equal to the force per unit mass applied to the particle.*

In other words, if \mathbf{F} represents the (vector) sum of all forces acting on a particle of mass m , any inertial observer will see that the particle has an acceleration \mathbf{a} which is given by,

$$\mathbf{F} = m\mathbf{a} . \tag{1}$$

This equation introduces two new concepts: force and mass. Precise definitions for these concepts are not easy even though we all have some intuition about both force and mass. Forces result when bodies interact. Forces are vectors with magnitude and direction. They are measured by comparison, e.g. to the weight of a standard mass, or to the deformation of a spring. We will assume that the concept of force is absolute and does not depend on the observer. Once we have defined force, we can define mass as the constant of proportionality between force and acceleration. Mass is a scalar quantity. One could think of mass as the resistance of bodies to a change in motion. That is, a given force applied to a body with small mass will produce a large acceleration, whereas the same force on a body of large mass will produce a small acceleration.

Equation (1) is a vector equation. This means that the force and the acceleration always have the same direction and the ratio of their magnitudes is m . It turns out that this equation is the basis for most engineering dynamics.

3.- *The forces of action and reaction between interacting bodies are equal in magnitude and opposite in direction*

This law makes explicit the fact that a force is the result of interaction between bodies. This law is clearly satisfied when the bodies are in contact and in static equilibrium. The situation for bodies in motion interacting at a distance, e.g. electromagnetic or gravity interactions, is a little bit more complicated. We know that electromagnetic signals travel at a finite speed and therefore there is a time delay whenever two bodies interact at a distance. Unfortunately, Newton did not foresee such a situation, and in these cases, Newton's third law breaks down. However, for practical purposes and for most engineering applications, the error made by assuming that these interactions are instantaneous, and, hence, assuming that Newton's third law is applicable, is negligible.

Note

Units

We shall use primarily two systems of units: the *International System*, also called SI, and the *English System*. The international system is the most widely used system for science and engineering. The English system, however, is still in widespread use within the United States engineering community. For this reason we will use and should feel comfortable with both systems. The SI units are *meter* (m), *kilogram* (kg), and *second* (s) for length, mass and time, respectively. Acceleration is measured in m/s^2 , and force is measured in $\text{kg}\cdot\text{m}/\text{s}^2$, which is also called a *Newton* (N) i.e. $1\text{N} = 1\text{Kg}\cdot\text{m}/\text{s}^2$. In the English system, the units of length, mass and time are *foot* (ft), *slug*, and *second* (s), respectively. Acceleration is measured in ft/s^2 , and force

is measured in slug·ft/s², also called *pound*, i.e. 1 pound = 1slug·ft/s². We have the following conversion factors:

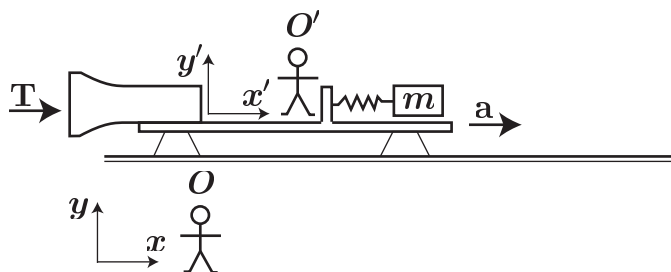
$$\begin{aligned} 1 \text{ ft} &= 0.348 \text{ m} \\ 1 \text{ slug} &= 14.5 \text{ kg} \\ 1 \text{ N} &= 0.224 \text{ lb} \end{aligned}$$

Example

Inertial vs. Non-inertial observers

This example is meant to illustrate the fact that we can easily come up with situations in which Newton's second law is not satisfied for accelerating, non-inertial observers. We will come back to this example later on in the course.

Consider a rocket sled which can move on a horizontal track as shown. We consider an inertial observer O which is fixed on the ground and an observer O' which is on the sled. We also have an accelerometer mounted on the sled. This consists of a known proof mass m , whose horizontal motion relative to the sled is constrained by a spring. We assume that the friction between the mass and the sled is negligible. This means that the only mechanism to exert a horizontal force to the mass is through the spring.



We consider two situations:

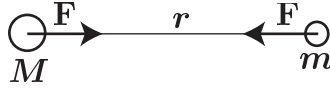
- The engine is off, $\mathbf{T} = \mathbf{0}$, $\mathbf{a} = 0$, the mass is at rest and the spring is uncompressed. Both observers O and O' agree in all their measurements.
- The engine is on, \mathbf{T} is a constant non-zero force, the sled has an acceleration \mathbf{a} , and the spring is compressed and exerting a force \mathbf{F} on the mass. Both observers are able to measure the force exerted by the spring by reading on a scale how much the spring deforms. For observer O' , the mass is not moving (assume that the initial transient oscillation is gone and the spring has settled to a fixed position relative to the sled). For the observer on the ground the mass is accelerating with acceleration \mathbf{a} . Since observer O is an inertial observer, he or she is able to verify that $\mathbf{F} = m\mathbf{a}$. On the other hand, the observer on the sled, O' , measures a force \mathbf{F} , but observes a zero acceleration. Hence, the observations of observer O' do not satisfy Newton's second law.

Law of Universal Attraction

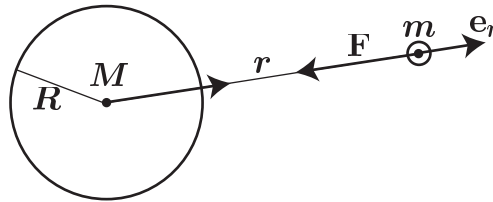
The law of universal attraction was proposed by Newton shortly after formulating the laws of motion. The law postulates that the force of attraction between any two particles, of masses M and m , respectively, has a magnitude, F , given by

$$F = G \frac{Mm}{r^2} \quad (2)$$

where r is the distance between the two particles, and $G = 6.673(10^{-11}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$ is the universal constant of gravitation which is determined according to experimental evidence. The direction of the force is parallel to the line connecting the two particles.



The law of gravitation stated above is strictly valid for point masses. One would expect that when the size of the masses is comparable to the distance between the masses one would observe deviations to the above law. It turns out that if the mass M is distributed uniformly over a sphere of radius R , the force on a mass m , *outside* M , is still given by (2), with r being measured from the sphere's center.



Weight

The gravitational attraction from the earth to any particle located near the surface of the earth is called the weight. Thus, the weight, \mathbf{W} , of a particle of mass m at sea level is given by

$$\mathbf{W} = -G \frac{M_e m}{R_e^2} \mathbf{e}_r = -g_0 m \mathbf{e}_r = m \mathbf{g}_0 .$$

Here, $M_e \approx 5.976 \times 10^{24} \text{ kg}$ and $R_e \approx 6.371 \times 10^6 \text{ m}$, are the mass and radius of the earth, respectively, and $\mathbf{g}_0 = -(GM_e/R_e^2) \mathbf{e}_r$, is the *gravitational* acceleration vector at sea level. The average value of its magnitude is $g_0 = 9.825 \text{ m/s}^2$.

The variation of the gravitational attraction with altitude is easily determined from the gravitational law. Thus, the weight at an altitude h above sea level is given by

$$\mathbf{W} = -G \frac{M_e m}{(R_e + h)^2} \mathbf{e}_r = -g_0 \frac{R_e^2}{(R_e + h)^2} m \mathbf{e}_r = m \frac{R_e^2}{(R_e + h)^2} \mathbf{g}_0 .$$

It turns out that the earth is not quite spherical and so the weight does not exactly obey the inverse-squared law. The magnitude of the gravitational acceleration, g_0 , at the poles and at the equator, is slightly different. In addition, the earth is also rotating. As we shall see this introduces an inertial centrifugal force which has the effect of reducing the vertical component of the weight. We will study these effects later on in the course.

References

- [1] R. Dugas, *A History of Mechanics*, Dover, 1988.

ADDITIONAL READING

J.L. Meriam and L.G. Kraige, *Engineering Mechanics, DYNAMICS*, 5th Edition
1/1, 1/2, 1/3, 1/4, 1/5 (Effect or Altitude only), 1/6, 1/7, 3/2