# 16.31 Homework Assignment #3

1. Given the plant $G(s) = 1/s^2$, design a lead compensator so that the dominant poles are located at $-1 \pm 1j$

2. Determine the required compensation for the system

$$G(s) = \frac{K}{(s+8)(s+14)(s+20)}$$

to meet the following specifications:

- Overshoot $\leq 10\%$
- 10-90% rise time $t_r \leq$100 msec

Simulate the response of this closed-loop system to a step response. Comment on the steady-state error. You should find that it is quite large.

Determine what modifications you would need to make to this controller so that the system also has

- $K_p > 6$

thereby reducing the steady state error. Simulate the response of this new closed-loop system and confirm that all the specifications are met.

3. Develop a state space model for the transfer function (not in modal/diagonal form). Discuss what state vector you chose and why.

$$G_1(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)} \tag{1}$$

(a) Develop a "modal" state space model for this transfer function as well.

(b) Confirm that both models yield the same transfer function when you compute

$$\hat{G}(s) = C(sI - A)^{-1}B + D$$

4. A set of state-space equations is given by:

$$\begin{aligned}\dot{x}_1 &= x_1(u - \beta x_2)\\ \dot{x}_2 &= x_2(-\alpha + \beta x_1)\end{aligned}$$

where $u$ is the input and $\alpha$ and $\beta$ are positive constants.

(a) Is this system linear or nonlinear, time-varying or time-invariant?

(b) Determine the equilibrium points for this system (constant operating points), assuming a constant input $u = 2$.

(c) Near the positive equilibrium point from (b), find a linearized state-space model of the system.