16.31 Homework Assignment #6

1. Consider the simple system

$$G(s) = \frac{s - z_o}{(s+3)(s+4)}$$

   (a) Confirm that one possible state-space model is given by:

$$A = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -z_o \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

   (b) Evaluate the controllability of this system as a function of the value of z_o.

   (c) Use pole placement techniques to derive the full-state feedback gains

$$u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   necessary to place the closed-loop poles at the roots of:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

   (d) Use your analytic expressions for k_1 and k_2 to support the following two claims:

   - "The system has to work harder and harder to achieve control as the controllability is reduced."
   - "To move the poles a long way takes large gains."

2. Given the MIMO system,

$$G(s) = \begin{bmatrix} \dfrac{6s+5}{s^2+3s+2} & \dfrac{7}{s+2} \\ \dfrac{5}{s+2} & \dfrac{6s+17}{s^2+5s+6} \end{bmatrix}$$

   Develop a state space model using:

   (a) The technique described at the bottom of page 10–5

   (b) Gilbert's realization method on page 10–6/7

   Confirm that the two state space models give the same transfer function matrix.

3. For the system

$$G(s) = \frac{s + 3.5}{(s + 1)(s + 2)}$$

(a) Design a compensator using classical techniques that results in a dominant response with a (5%) settling time (see 14–1) of approximately 0.75 sec and a maximum peak overshoot in the step response of about 5%.

(b) Develop a state space model of $G(s)$ and design a dynamic output feedback controller using the techniques given in class. Does the closed-loop system meet the specs?

(i) Use Ackerman's formula to design the regulator to place the poles to meet the design requirements given above (should be near $-4 \pm 4i$) – show your calculation by hand and confirm in Matlab.

(ii) Use Matlab to place the estimator poles so that the real part is twice that of the regulator poles.

(iii) Combine the regulator and estimator to form the compensator $G_c(s)$. Does the closed-loop system meet the specs?

(c) Compare your design with the dynamic output feedback compensator - use both the frequency and time response to make this comparison.

4. We have talked at length about selecting the feedback gains to change the pole locations of the system, but we have not mentioned anything about how the open-loop zeros are changed. Part of the reason for this is that it can be shown that:
When full state feedback is used ($u = \bar{N}r - Kx$) to control a system, the zeros remain unchanged by the feedback.

Confirm that this statement is true by analyzing the zero locations for the closed-loop system, which are given by the roots of the polynomial:

$$\det \begin{bmatrix} sI - (A - BK) & -\bar{N}B \\ C & 0 \end{bmatrix} = 0$$

The best way to proceed is to show that, through a series of column and row operations that **do not** change the value of the determinant, you can get the following reduction:

$$\det \begin{bmatrix} sI - (A - BK) & -\bar{N}B \\ C & 0 \end{bmatrix} = 0 \quad \Rightarrow \quad \det \begin{bmatrix} sI - A & -B \\ C & 0 \end{bmatrix} = 0$$

which, of course, is the same polynomial used to find the open-loop zeros of the system.