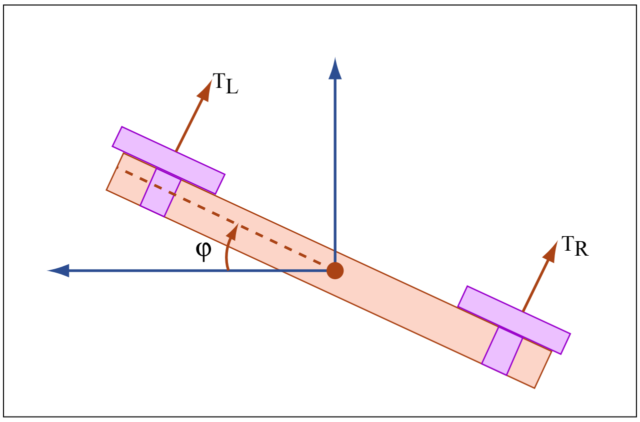
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.31: Feedback Control Systems
Fall 2007

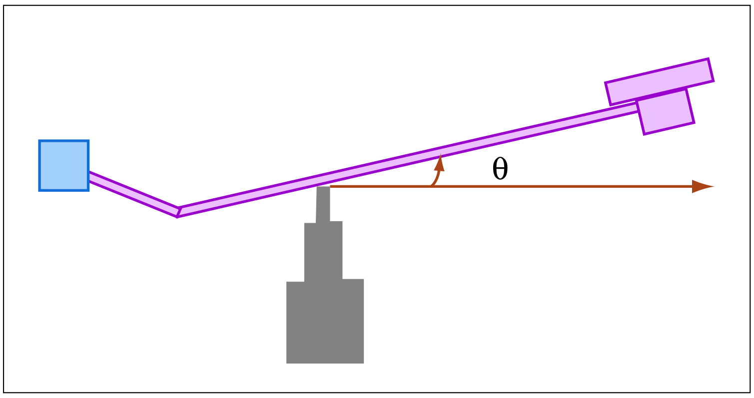
Lab 1: Dynamics and Basic Control
Due: October 19th, 2007 (5pm)

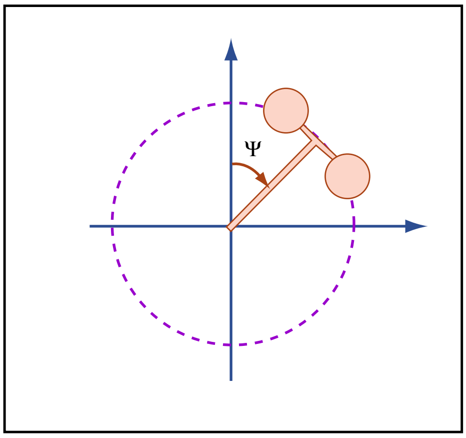
1 Introduction

The Quanser helicopter is a mechanical device that emulates the flight of a reduced degree of freedom (DOF) helicopter. Instead of the usual six DOF of a free-flying helicopter, the Quanser only exhibits three: roll ϕ, pitch θ, and travel ψ, as illustrated in Figures 1-3. The Quanser system is actuated by two rotor speeds and the inputs to the system are V_{cyc}, which is an electric voltage that results in differential change in the two rotor speeds, and V_{col}, which is an electric voltage that controls the speed of the two propellers collectively. The outputs of the system are three angles: roll ϕ, pitch θ, and travel ψ.

Please note the limits of the Quanser: voltage $\in [-5, 5]$ Volts, $\phi \in [-40°, 40°]$, and $\theta \in [-25°, 30°]$.







Figures by MIT OpenCourseWare.

2 Physical Model

The Quanser model is derived by applying Newtons second law to the rate of change of angular momentum. Equations 1-3 are the nonlinear equations of motion:

$$I_{yy}\ddot{\theta} = \tau_{coll} l_{boom}\cos(\phi) - Mgl_{\theta}\sin(\theta+\theta_{rest}) - Dl_{boom}\sin(\gamma) + I_r\omega_{rotor}\dot{\phi} \quad (1)$$
$$I_{xx}\ddot{\phi} = \tau_{cyc} l_h - mgl_{\phi}\sin(\phi) + L_p\dot{\phi} - I_r\omega_{rotor}(\dot{\theta}\cos(\phi) + \dot{\psi}\sin(\phi)) \quad (2)$$
$$I_{zz}\ddot{\psi} = \tau_{coll} l_{boom}\sin(\phi) - Dl_{boom}\cos(\gamma) \quad (3)$$

We can then linearize these equations around a steady-state hover operating condition, i.e. $\dot{\phi} = \dot{\theta} = \dot{\psi} = 0$, $\phi = \theta = \psi = 0$, $\gamma = 0$ (flight path angle), $v_0 = 0$ to get:

$$I_{yy}\ddot{\theta} = (K_\tau\omega_{coll} - K_v\dot{\psi})l_{boom} - M_q\dot{\theta} - [Mgl_\theta\cos(\theta_{rest})]\theta \quad (4)$$
$$I_{xx}\ddot{\phi} = (K_\tau\omega_{cyc} - K_v\dot{\psi})l_h - [mgl_\phi]\phi - L_p\dot{\phi} \quad (5)$$
$$I_{zz}\ddot{\psi} = [\tau_{coll} l_{boom}]\phi - [K_D l_{boom}\cos(\gamma_0)]\dot{\psi} \quad (6)$$

In addition, the motor dynamics can be modeled by

$$\dot{\omega}_{cyc} + 6\omega_{cyc} = 780V_{cyc} \quad (7)$$
$$\dot{\omega}_{coll} + 6\omega_{coll} = 540V_{coll} \quad (8)$$

Based on these linearized equations, the pitch (θ) and roll (ϕ) subsystems can be represented by the block diagrams

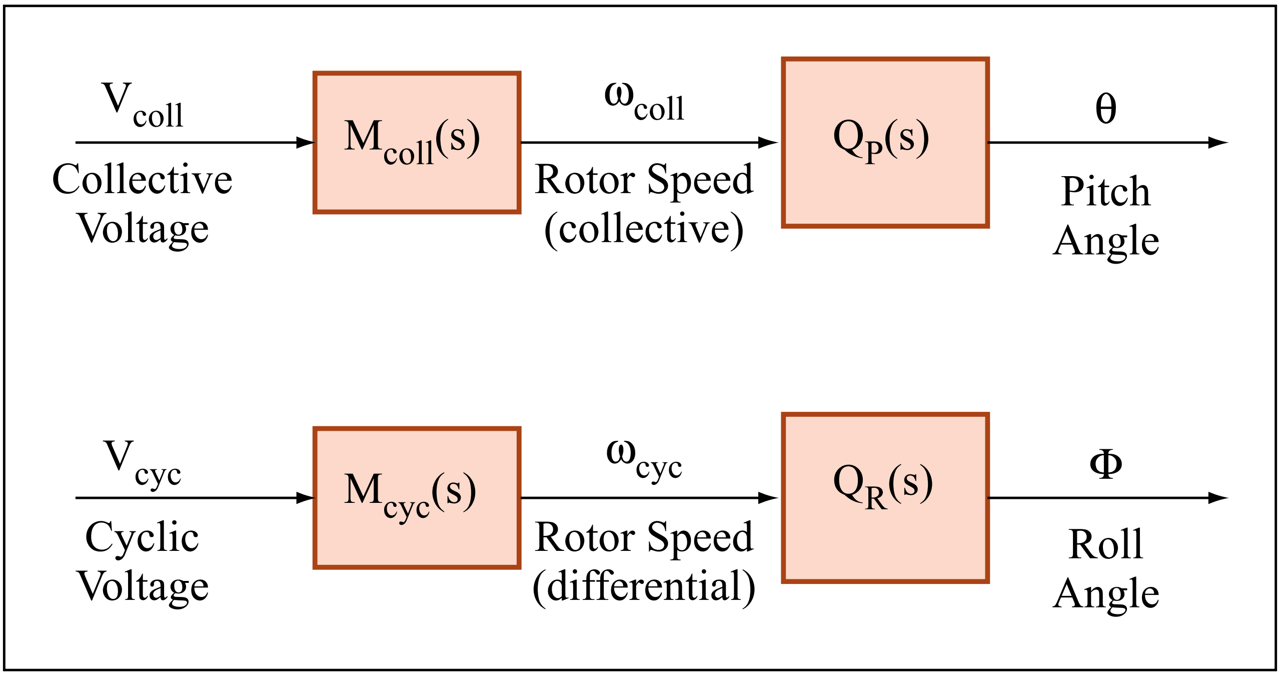


Figure by MIT OpenCourseWare.

Where $M_{coll}(s)$ is the collective voltage to collective rotor speed transfer function:

$$M_{coll}(s) = \frac{\omega_{coll}(s)}{V_{coll}(s)} = \frac{540}{s+6} \quad (9)$$

$Q_P(s)$ is the collective rotor speed to pitch angle transfer function:

$$Q_P(s) = \frac{\Theta(s)}{\omega_{coll}(s)} = \frac{K_\tau l_{boom}}{I_{yy}s^2 + M_q s + Mgl_\theta\cos(\theta_{rest})} \quad (10)$$

$M_{cyc}(s)$ is the cyclic voltage to cyclic rotor speed transfer function:

$$M_{cyc}(s) = \frac{\omega_{cyc}(s)}{V_{cyc}(s)} = \frac{780}{s+6} \quad (11)$$

$Q_R(s)$ is the cyclic rotor speed to roll angle transfer function:

$$Q_R(s) = \frac{\Phi(s)}{\omega_{cyc}(s)} = \frac{K_\tau l_h}{I_{xx}s^2 + L_P s + mgl_\phi} \tag{12}$$

Table 1: Physical parameters

Parameter	Value	Units	Description
m	1.15	kg	mass of rotor assembly
M	3.57	kg	mass of whole setup
l_{boom}	0.66	m	length from pivot point to heli body
l_ϕ	0.004	m	length of pendulum for roll axis
l_θ	0.014	m	length of pendulum for pitch axis
l_h	0.177	m	length from pivot point to the rotor
I_{xx}	0.036	Nm	moment of inertia about x-axis
I_{yy}	0.93	Nm	moment of inertia about y-axis
K_τ	4.25×10^{-3}		coefficient of thrust
θ_{rest}	-25	degs	theta rest value
L_p	$[0.02, 0.2]$		roll damping coefficient
M_q	$[0.1, 0.9]$		pitch damping coefficient

The linear model presented above does not perfectly represent the dynamics of the actual Quanser for many reasons. While there are many inaccuracies in our model, the variations in pitch and roll damping from Quanser to Quanser present the largest problem. You should assume that the damping coefficients L_p and M_q for any given Quanser will be somewhere in the ranges listed in Table 1.

3 PreLab Assignment

1. Use classical design techniques to develop controllers for the roll (ϕ) and pitch (θ) axes. You are encouraged to use Matlab/Sisotool to assist you in your design. Remember that you are designing the controllers to work for the entire range of damping constants L_p and M_q. For the nominal plants (i.e., use a damping value in the middle of the range), your controllers must have the following properties:

   - Less than 10% error in tracking for signals with frequency content up to 1 rad/s.
   - At least 20° of phase margin
   - Maintain stability even for the worst-case damping

   Hint: Your roll (ϕ) controller is going to be the "inner-loop" for the travel controller in the next lab, i.e. the travel controller will be commanding roll angle. Thus it is important that your roll controller is fast and stable but not so important that it has zero steady-state error. Conversely, your pitch (θ) controller should have zero steady-state error so that you are sure to clear altitude obstacles in the next lab.

   For both of your controllers, write the controller transfer function and plot the following:

   - Bode plot of the controller
   - Open loop Bode plot of controller + nominal plant
   - Simulated step response for a 20° step command
   - A paragraph or two describing your design strategy, predicted performance, and pros/cons of your design

2. Analyze the stability and performance of your controller for the entire range of L_p and M_q values in Table 1).

You should have your completed prelab assignment with you to be checked off in order to start the lab. You must also hand in the prelab assignment with your lab writeup, and it will count for a substantial portion of the overall lab grade.

4 Lab Procedure

1. Lab sessions will be over the next 2 weeks. Sign-up sheets will be available in lecture, and it is your responsibility to sign up for a lab time. Due to the class size, we will have to work in groups of 3–4 people. Please select partners and email me your group by Wednesday of next week. Anyone not in a group by noon on Wednesday will be assigned to a group.
Each member of the group must complete prelab assignment and you must submit your own lab report.
2. Specific instructions on how to construct your controller and build your model in Matlab will be posted in the lab. We will try to arrange for some support in the lab if you get stuck - but recognizing that there is no TA for the class, this feedback process might take a while, so plan ahead.
3. Apply steps of appropriate amplitude and duration to record system output in roll and pitch. Compare with the expected response from your models, and redesign your controller gains if necessary.
4. Record and save the following step responses (to be included in your lab writeup):
   - Roll step input of 20° with $\phi_0 = 0°$; $\theta = 0°$ and 10°
   - Pitch step input of 20° with $\theta_0 = 0°$; $\phi = 0°$ and 20°
5. Once you are happy with the performance of your controllers on your Quanser, save your step responses and record your final controller transfer functions. Make sure you either email the data to yourself or use SecureFX to upload it the MIT Server so that you can access it when you write your lab report.

5 Lab Writeup

The lab writeup need not be particularly formal. However, you MUST include the following to receive full credit:

1. Your controller transfer functions
2. The recorded step responses listed in "Lab Procedure" WITH the corresponding theoretical responses (from your model) overlayed in the same plot. Give a brief description of the properties of each observed response (i.e. overshoot, rise time, etc.).
3. A few paragraphs discussing how well the theoretical responses from the model matched the actual responses you measured in the lab. Give reasons for any inconsistencies and explain the importance of accounting for the plant uncertainty.
4. Plots showing the results of your flight test. Discuss how robust your controller proved to be. List some ideas that might make your controller more robust.

Please observe standard technical writing conventions (i.e. label all charts and figures, put data in tables, etc.).