Massachusetts Institute of Technology

16.410-13 Principles of Autonomy and Decision Making

Assignment #4 (written part) **Due: Session 9**

Objective

To become comfortable framing problems as Linear Programming Problems, to practice converting problems into a form suitable for solution using the Simplex Method. These problems require that you read the background reading (see below). You are to solve the problems by hand (not using an LP solver). You are expected to write out the steps of your solutions. You may at your discretion use the algebraic or the tabular techniques described in the chapter.

There is no programming part to this problem set.

Background

Solving Linear Programming Problems: The Simplex Method (handed out in class). This is Chapter 4 of "Introduction to Operations Research" by Frederick S. Hillier, Gerald J. Lieberman.

Problem 1: Solve LP problem

Consider the following problem:

Maximize $Z=x_1 + 2x_2$
subject to $x_1 + 3x_2 \leq 8$
$x_1 + x_2 \leq 4$
$x_1 \geq 0$
$x_2 \geq 0$

Part a: Solve this problem graphically. Identify all corner-point feasible solutions.
Part b: Solve this problem (by hand) using the simplex method.

Problem 2: Solve LP problem

Solve the following linear program using Simplex.

Minimize $z = 8x_1 + 3x_2 + 18x_3 + 20x_4$

subject to $x_1 + x_2 \geq 8$
$x_1 \leq 15x_3$
$x_2 \leq 10x_4$
$x_3 + x_4 \leq 1$

Problem 3: Formulate and Solve LP

A jeweler can make three kinds of necklaces, red, green, and mixed color. The red necklaces sell for $4, the green necklaces sell for $2 and the mixed color necklaces sell for $3. Each necklace consists of 100 colored beads. The mixed color necklaces are made from 50 red beads and 50 green beads. The jeweler must not make more than 500 red necklaces in a day. The jeweler receives 800 red beads and 600 green beads a day and he wants to maximize his daily income from necklace making. All beads must be paid for and unused bead at the end of the day are discarded. Red beads cost $10 for 800 and green beads cost $6 for 600. How many red, green, and mixed color necklaces should he make?

Part a: Formulate the above problem as a linear programming problem.
Part b: Solve the linear programming problem using Simplex.

1) What is the maximum daily profit?
2) How many red necklaces will he make per day?
3) How many green necklaces will he make per day?
4) How many mixed colored necklaces will he make per day?

Part c: Can you add a constraint that the jeweler makes at least 2 of each kind of necklace each day? If so what complications does this add and how are those complications overcome? Either solve this extended problem or explain why it cannot be solved.

Problem 4: Solve LP problem

Consider the following problem.

Minimize $Z = 3x_1 + 2x_2$,
Subject to:

$2x_1 + x_2 \geq 10$
$-3x_1 + 2x_2 \leq 6$
$x_1 + x_2 \geq 6$
$x_1, x_2 \geq 0$

Part a: Solve this problem graphically.
Part b: Solve using the simplex method. Describe your method.

Problem 5: Time

Please let us know the amount of time it took you to complete this problem set.