Massachusetts Institute of Technology

16.410 Principles of Automated Reasoning and Decision Making

Problem Set #8 **Due:Session18**

Propositional Logic

Objectives

In this problem set we develop your understanding of logical representation and reasoning. First, you will learn to count the number of models that satisfy a given sentence. Next, you will determine the satisfiability of a sentence using truth tables and equivalence rules. Finally, you will derive the truth of a statement based on a knowledge-base of sentences.

Readings

AIMA Chapter 7.

Problem 1 – Planner Implementation (40 pts in Pset #7).

Submit your solution to Problem 3 “Cleaning up Rooms” described in Problem Set #7. Recall that this problem involved encoding a planning problem and executing it on a planner.

Problem 2 – Problem #7.5, AIMA (15 points)

In this problem, you will consider a vocabulary with four propositions, A, B, C, and D. How many models are there for the following sentences? Justify each answer.

a.) $(A \wedge B) \vee (B \wedge C)$
b.) $A \vee B$
c.) $A \Leftrightarrow B \Leftrightarrow C$

Problem 3 – Problem #7.8, AIMA (25 points)

Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 in AIMA.

a.) $Smoke \Rightarrow Smoke$
b.) $Smoke \Rightarrow Fire$
c.) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
d.) $Smoke \vee Fire \vee \neg Fire$
e.) $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$
f.) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$
g.) $Big \vee Dumb \vee (Big \Rightarrow Dumb)$
h.) $(Big \wedge Dumb) \vee \neg Dumb$

Problem 4 – Problem #7.9, AIMA (25 points)

Restate the following English in propositional logic. Given this sentence encoding, can you prove that the unicorn is mythical? How about magical? Horned? Please include your proof for each.

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Problem 5 – Propositional Logic and Inference (30 points)
(From 16.410/13 Final, Fall 04)

Part A – Interpretations (3 points)

Let S be the propositional sentence:

(A implies B) implies (C implies D)

and let I be the interpretation:
A = True, B = False, C= True, D = False

Is I a model for S, that is, does I satisfy S (Yes or No)?

Demonstrate the correctness of your answer in the following box:

Part B – Reduction to Clauses (Conjunctive Normal Form) (9 points)

Reduce the following three propositional sentences to conjunctive normal form (CNF) (i.e., a set of clauses). Derive each result step by step:

Part B.1 **Convert [not ((not A and B) or (C and D))] to CNF**

Part B.2 **Convert (A iff A) to CNF**

Part B.3 **Convert [(A iff B) or C] to CNF**

Part C – Satisfiability Using DPLL (8 points)

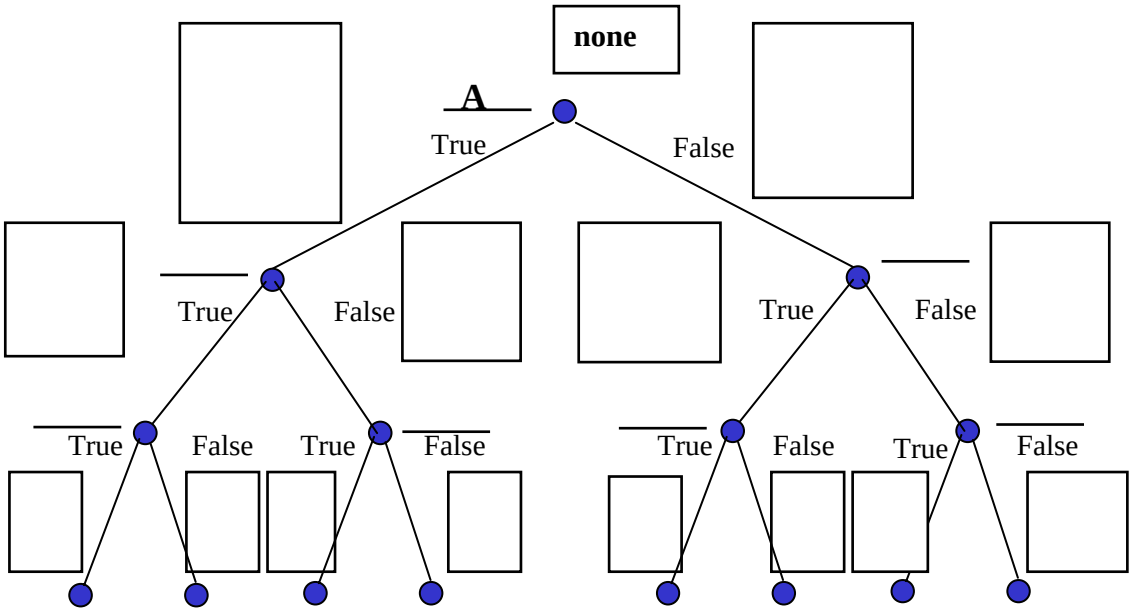
Consider a theory comprised of the following seven clauses:

A or B or C;
A or E or F;
not B or not D or E;
not C or D;
not A or B;
not B or F;
not E or not F;

Use the DPLL algorithm (backtrack search plus unit propagation) to find ALL truth assignments **to propositions A, B, C, D, E and F, that are consistent with the theory. Fill out the** search tree supplied below. **You must** explore the complete search tree.

- Search **the propositions** in alphabetical order **(no other order please!).**
- **For each proposition P,** assign **the value** True before **trying** False.
- **On the line next to each tree node,** write the proposition being assigned **a truth value at that point in the search.**
- **In the box** next to each branch, list each proposition **whose truth value is** determined by unit propagation, **based on the assignment to the proposition at that branch.**
- Indicate **the** truth value derived **for each of these propositions.**
- Draw an X at each node **that is immediately below the branch** where at least one clause becomes false; **this is where the search backtracks.**
- Circle **the node that denotes the** first complete and consistent assignment **to the propositions A – F.**

We filled out the result of the initial propagation in the box above the tree. In addition, we filled out the first variable to be assigned (A), next to the root.



Problem 6: Time (5 pts)

Please let us know the amount of time it took you to complete this problem set. Please separate the amount of time for the written and for the coding components.

Note that we do track these loads on a weekly basis.