Let's consider the Fixed Arrival Time problem.

We want to find a minimum fuel solution that takes our craft from to starting (x, y) position to the target (x, y) position so that it arrives at (exactly) time T and we want to get there in N time steps each of time Δt. T/N=Δt. There is no notion of negative fuel so we can instead think in terms of thrust in four dimensions ForwardThurstX (ft_x), BackwardThrustX(bt_x), ForwardThrustY(ft_y) and BackwardThrustY(bt_y). This is by no means the only way of representing this information—it is just an example designed to be easy to understand.

Therefore our objective function is to minimize J_T as follows:

$$\min_{U_i} = \min_{U_i} \sum_{i-0}^{N-1} Cost^T U_i$$

We will assume that thrust in each direction burns the same amount of fuel so:

$$Cost^T = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$$

Where $U_i = \begin{pmatrix} ftx_i \\ btx_i \\ fty_i \\ bty_i \end{pmatrix}$

We are solving to get a sequence of N states $S_0 \ldots S_{N-1}$ resulting from a sequence of N control inputs $U_0 \ldots U_{N-1}$. The state needs to specify both position and velocity or the two dimensions of the problem. We must consider the fact that simplex requires all variables to be >=0. There are several solutions to this problem. We will make the assumption that our boat is limited to a 2000x2000 region of space and set the origin in the center by replacing x_n with x'_n-1000 and similarly for y. For velocities we could do a similar thing with bounded velocities or we could handle unbounded +ve or –ve velocities by adding variables (see page 92 of the Introduction to Operations Research). One particularly simple solution is to keep +ve (forward) and –ve (backward) velocities separate in the state representation as follows.

$S_n = \begin{pmatrix} x'_n \\ y'_n \\ fvx_n \\ bvx_n \\ fvy_n \\ bvy_n \end{pmatrix}$ with constraints on x'_n and y'_n as follows: $\begin{matrix} x'_n < 2000 \\ x'_n \geq 0 \\ y'_n < 2000 \\ y'_n \geq 0 \end{matrix}$

Now, time step is related to the previous time step by the dynamics constraints.

$$S_{i+1} = A\ S_i + B\ U_i$$

Where A updates the position based on the velocity and the velocities based on drag while B implements the acceleration due to fuel burn specified in the control U.

$$A = \begin{pmatrix} 1 & 0 & \Delta t & -\Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & -\Delta t \\ 0 & 0 & r\Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & r\Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & r\Delta t & 0 \\ 0 & 0 & 0 & 0 & 0 & r\Delta t \end{pmatrix}$$ where (1-r) is the water resistance.

Thrust w=ma, so acceleration a=w/m where w=efficiency*fuel

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{efficiency}{m} & & 0 & 0 \\ 0 & \frac{efficiency}{m} & & 0 \\ 0 & 0 & \frac{efficiency}{m} & 0 \\ 0 & 0 & 0 & \frac{efficiency}{m} \end{pmatrix}$$

We need a constraint that the final state is the goal state:

$$S_{N+1} = \begin{pmatrix} x'_G \\ y'_G \\ fvx_G \\ bvx_G \\ fvy_G \\ bvy_G \end{pmatrix}$$

We also need a constraint that the start state is the start state

$$S_0 = \begin{pmatrix} x'_S \\ y'_S \\ fvx_S \\ bvx_S \\ fvy_S \\ bvy_S \end{pmatrix}$$

The fixed arrival time problem can be solved by unrolling the above equations and solving using the Simplex solver. Subtracting out 1000 from each of x'_i and y'_i will yield values for x_0 y_0, x_1 y_1, …, x_{N-1} y_{N-1}. Similarly the fuel flow at each step can be calculated by adding for each step i $ftx_i + fty_i + btx_i + bty_i$

Additional constraints can, optionally, be added that put limits of the maximum velocity.