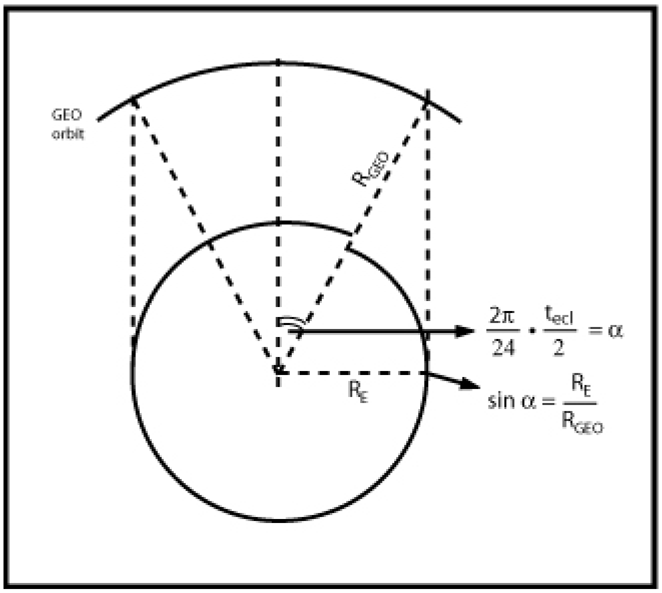
Lecture 6: Hydrazine Decomposition: Performance Estimates

(3) Electrothermal Augmentation Concept

Geostationary satellites are most of the time exposed to the sun, but they still are subject to eclipse periods around the two vernal points (March 21, September 21); when the intersection of the orbital plane (Equator) and the ecliptic plane points to the sun. The maximum eclipse length is



$$t_{ecl.} = 2\frac{24}{2\pi}\sin^{-1}\left(\frac{R_E}{R_{GEO}}\right) = 1.16\,\text{hr}$$

(once per day in the "eclipse season")

If the satellite is to remain active during these occultations, enough battery capacity must be incorporated.

But obviously, these batteries have ample time to recharge even in eclipse season, and are idle most of the time.

Starting from the payload requirements, the solar array capacity at End of Life (EOL) is typically dimensioned with a 15% margin to allow for battery charging losses. A further 15% is then added (depending on mission duration and altitude) to allow for array degradation from Beginning of Life (BOL) to EOL. An example is the military satellite DSCS III:

Array output (BOL/EOL)	977 watt/837 watt ($28V \pm 1\%$)
Batteries	750 watt
	1968 watt-hr (100% DoD) [depth of discharge]
	1180 watt-hr (60% DoD)
Payload requirements	723 watt

Thus, there is the possibility of using occasionally both the batteries and the arrays to provide power for overheating the gas generated by a N_2H_4 decomposition chamber and increase the performance. In the case of the DSCS III, the power available for electrothermal augmentation (ETA) is

BOL	EOL
977 w array	837 w array
+750 w batteries	+750 w batteries
-723 w payload	-723 w payload
1004 w for ETA	864 w for ETA

If the firing is for 1 hr, the batteries provide 750 watt-hr, which is 38% depth of discharge only, and should not compromise battery life.

To calculate the performance achievable, we can use the enthalpy equation again; this time, since the temperature is going to be high and the residence time in the heater long, we can assume equilibrium is reached, which essentially means x=1 (all NH_3 decomposed)

$$N_2H_4 \rightarrow N_2 + 2H_2 \qquad\qquad (+0.5\%\ H_2O + \sim 0.5\%\ NH_3)$$

Thus

$$h_{gas}\left(\text{at T}\right)\left(\text{Kc per 32 grams}\right) = \left(-2.83 + 7.75\,\theta + 0.183\,\theta^2\right) + 2\left(-1.967 + 6.6\,\theta + 0.367\,\theta^2\right)$$

$$c_p\left(T, \text{cal/g}^o\text{C}\right) = \frac{1}{32}\frac{d\,h_{gas}}{d\theta} = +0.6547 + 0.0573\,\theta$$

$$c_v(T, \text{cal/g}^o\text{C}) = c_p - \frac{1.987}{10.67} = 0.4685 + 0.0573\,\theta$$

$$\gamma(T) = c_p / c_v \qquad\qquad \begin{array}{l} \theta = 2 \rightarrow r = 1.319 \\ \theta = 1.4 \rightarrow r = 1.335 \end{array}$$

$$M = \frac{28 + 2x2}{3} = 10.67 \text{ g/mole}$$

Given the available electrical energy (E) per Kg of gas (or power/mass flow rate), we can now solve for θ by setting

$$E\left(\frac{\text{Joule}}{\text{Kg}}\right) = \left[h_{gas}\left(\text{Kcal/32g.}\right) - \underbrace{12.0}_{\text{liquid } N_2H_4 \text{ at } 298°K}\right] x\overbrace{130,800}^{\frac{4180\text{J/Kcal}}{0.032\text{Kg/mol}}}$$

We can also calculate the performance in the same way as done before (from γ, M, P_e/P_0). However, since c_p does vary with T, let us do it a bit better this time. Assuming the nozzle expansion is ideal,

$$ds = 0 \rightarrow dh = \frac{1}{\rho}dp \qquad c_p dT = \frac{R_g T}{P}dp$$

$$\frac{c_p(T)}{R_g} d\ln T = d\ln P$$

$$\frac{P}{P_0} = \exp\left(\int_{T_0}^{T} \frac{c_p(T)}{R_g} d\ln T\right)$$

Notice if c_p= constant, this gives

$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{\gamma}{\gamma-1}}, \text{ as usual.}$$

Now, since

$$c_p = a + bT,$$

$$\int_{T_0}^{T} c_p d\ln T = \int_{T_0}^{T}\left(\frac{a}{T}+b\right)dT = a\ln\frac{T}{T_0} + b\left(T - T_0\right)$$

With our values, this gives

$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{a}{R_g}} e^{\frac{b}{R_g}\left(T-T_0\right)} \quad ; \qquad \frac{a}{R_g} = \frac{0.6547\,\frac{cal}{g^oC}}{\left(\frac{1.986}{10.67}\right)\frac{cal}{g^oC}} = 3.515, \text{etc}$$

$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{3.515} e^{0.3075\,\frac{T-T_0}{1000}}$$

The nozzle is likely to be specified by a given area ratio rather than a given pressure ratio. To relate these two, let us first calculate the throat conditions. In general

$$\frac{\dot{m}}{A} = \rho u = \frac{P(T)}{R_g T}u = \frac{P(T)}{R_g T}\sqrt{2\underbrace{\left(h_0 - h\right)}_{\text{enthalpy per Kg}}}$$

At the throat A is minimum, so, setting

$$\frac{d\ln A}{dT} = 0,$$

$$-\frac{1}{T^*} + \left(\frac{d\ln P}{dT}\right)_{T^*} - \frac{\left(\frac{dH}{dT}\right)_{T^*}}{2\left(h_0 - h^*\right)} = 0$$

$$-\frac{1}{T^*}+\frac{c_p\left(T^*\right)}{R_gT^*}-\frac{c_p\left(T^*\right)}{2\left(h_0-h^*\right)}=0$$

$$\frac{\overbrace{c_p^*-R_g}^{c_v^*}}{R_gT^*}=\frac{c_p^*/2}{h_0-h^*}$$

$$\frac{h_0-h\left(T^*\right)}{R_gT^*}=\frac{1}{2}\gamma(T^*) \qquad \left(R_g=\frac{R}{M}\right)$$

Note:

$$\gamma^*=\frac{2\left(h_0-h^*\right)}{R_gT^*}=\frac{\left(u^2\right)^*}{R_gT^*}$$

So

$$\boxed{u^*=\sqrt{\gamma^*R_gT^*}} \qquad \text{(speed of sound \underline{at throat})}$$

(T^* = temperature at throat.)

Using the known functions h(T), γ(T), this equation can be solved for T^*.

If γ =constant and $h=c_pT$, this would give the usual

$$\frac{T^*}{T_0}=\frac{2}{\gamma+1}.$$

The area ratio now follows from the $\frac{\dot{m}}{A}$ expression:

$$\frac{A_e}{A^*}=\frac{T_e}{T^*}\frac{P^*}{P_e}\sqrt{\frac{h_0-h^*}{h_0-h_e}}$$

Given A_e/A^* and T_0, this equation can be solved for T_e, and hence P_e/P_0 can be calculated.

Finally, the usual performance parameters $c_F=F/P_0\,A^*$ and $c^*=\frac{P_0A^*}{\dot{m}}$ can be found:

$$c_F=\frac{\dot{m}u_e+P_eA_e}{P_0A^*}=\frac{\rho_eu_e^2A_e+P_eA_e}{P_0A^*}=\frac{P_e}{P_0}\frac{A_e}{A^*}\left(\frac{2\left(h_0-h_e\right)}{R_gT_e}+1\right)$$

$$c^* = \frac{P_0 A^*}{\dot{m}} = \frac{P_0}{P^*} \frac{R_g T^*}{\sqrt{2\left(h_0 - h^*\right)}}$$

From these,

$$I_{sp} = \frac{F}{\dot{m} g} = \frac{F}{P_0 A^*} \frac{P_0 A^*}{\dot{m}} \frac{1}{g} = \frac{c_F c^*}{g}$$, as usual.

Using an assumed area ratio $A_e/A^* = 50$, these calculations give the following results: (Notice the weak variation of c_F and P_e/P_0, due to the shifting γ. Also (but not shown), calculation using constant γ (using the γ^* for instance), gives very nearly the same results).

T_0($^\circ$K)(superheated gas temp.)	1200	1400	1600	1800	2000	2200	2400
T^*($^\circ$K) (temperature at throat)	1021	1194	1369	1544	1720	1897	2075
γ^*(at throat)	1.3536	1.3470	1.3407	1.3345	1.3286	1.3228	1.3172
h_0 (Kcal/mole N_2H_4)	19.693	24.36	29.1	33.913	38.8	43.76	48.793
P^*/P_0	0.5360	0.5370	0.5381	0.5392	0.5402	0.5412	0.5415
P_e/P_0	0.000758	0.000768	0.000778	0.000787	0.000796	0.000810	0.000821
T_e($^\circ$K)	170.0	202.1	235.3	269.4	304.5	341.7	379.5
c_F(thrust coefficient)	1.746	1.748	1.750	1.750	1.751	1.759	1.762
$c^*(\frac{m}{sec})$, (characteristic velocity)	1431	1548	1659	1766	1868	1954	2046
I_{sp}(sec)	255.0	267.3	296.2	315.3	333.7	350.9	367.8
$E=P_{elec.}/\dot{m}$ (MJ/Kg, augmentation power)	1.005	1.614	2.234	2.862	3.501	4.149	4.806

Performance of Electrothermally Augmented Hydrazine Microthrusters, as a Function of Ideal Augmentation Power. (For A_e/A^*=50, vacuum operation).

As we saw before, the thrust needed for NSSK depends on satellite mass, firing time and firing frequency

$$F = \frac{M}{t_b} v_c \frac{i}{\eta_{NSSK}} \qquad ; \qquad i = \frac{0.9^\circ}{N} \quad \begin{pmatrix} v_c = \text{circular velocity in GEO} \\ N = \text{no. of firings per year} \end{pmatrix}$$

From the engine viewpoint,

$$F = \dot{m} g I_{sp}$$

and if we have an electric power $P_{el.}$ and use it with an efficiency η (the rest are heat losses), the specific ETA energy is

$$E = \frac{\eta P_{el}}{\dot{m}} \rightarrow \dot{m} = \frac{\eta P_{el}}{E} \rightarrow F = \frac{\eta P_{el}}{E} g I_{sp}$$

where I_{sp} depends upon E (table).

For the DSCSIII satellite, assume $\eta = 0.75$ and BOL power (1004 watts). Suppose one can achieve a superheated gas temperature of 1800°K (mostly limited by materials).

Then, from the table,

$Isp = 315.3 \text{ sec}$,

$E = 2.862 \times 10^6 \text{J/Kg}$

So, the thrust is

$$F = \frac{0.75 \times 1004}{2.862 \times 10^6} 9.8 \times 315.3 = 0.8130 \text{ Nt}$$

This implies, for a DSCSIII mass of 1043 Kg

$$i = \eta_{NSSK} \frac{Ft_b}{Mv_c} = 0.9971 \frac{0.8130 \times 3600}{1043 \times 3071} = 9.111 \times 10^{-4} \text{rad} = 0.0522^\circ$$

and so

$$N = \frac{0.9}{0.0522} = 17.2 \text{ per year}$$

(every 21.2 days)