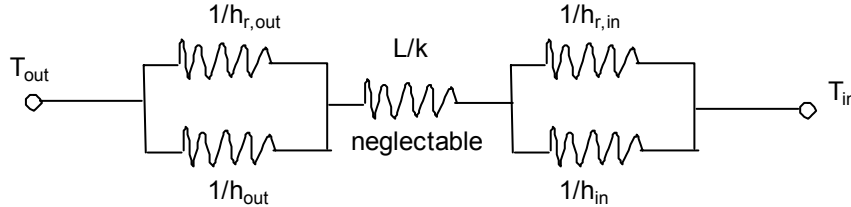


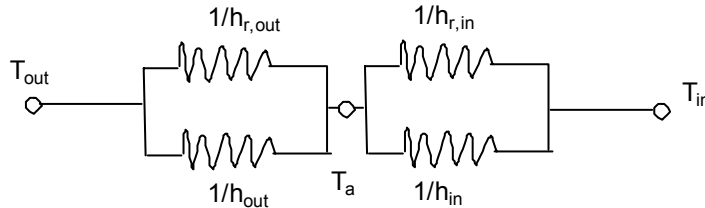
Solution for Quiz 2 of 2003

Problem 1

(a). The electric analogy for the heat transfer through the steel frame can be drawn as:



For steel material, its conductivity $k=15\text{W/m.K}$, the value of L/k for this steel frame is $0.02\text{m}/(15\text{ W/m.K})=0.0013\text{m}^2.\text{K/W}$. It is very small comparing with convection and radiation R values. So the conductivity R value of the steel frame can be neglected and the steel frame has a uniform temperature T_a . Then the electric analogy can be simplified:



(b). We can calculate the total R value of the steel frame as:

$$R = \frac{1}{1/(1/h_{r,out}) + 1/(1/h_{out})} + \frac{1}{1/(1/h_{r,in}) + 1/(1/h_{in})} = \frac{1}{h_{r,out} + h_{out}} + \frac{1}{h_{r,in} + h_{in}}$$

where,

$$h_{r,out} = 4\varepsilon\sigma\bar{T}^3 = 4 \times 1 \times 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 \times \left((273 + \frac{0+20}{2}) \text{ K} \right)^3 = 5.14 \text{ W/m}^2 \text{ K}$$

$$h_{r,in} = 4\varepsilon\sigma\bar{T}^3 = 4 \times 0.1 \times 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 \times \left((273 + \frac{0+20}{2}) \text{ K} \right)^3 = 0.51 \text{ W/m}^2 \text{ K}$$

Then the total R value of the steel frame can be gotten:

$$R = \frac{1}{h_{r,out} + h_{out}} + \frac{1}{h_{r,in} + h_{in}} = \frac{1}{5 \text{ W/m}^2 \text{ K} + 5.14 \text{ W/m}^2 \text{ K}} + \frac{1}{3 \text{ W/m}^2 \text{ K} + 0.51 \text{ W/m}^2 \text{ K}} = 0.38 \text{ m}^2 \text{ K/W}$$

The heat transfer through the steel frame q equals:

$$q = \frac{T_{in} - T_{out}}{R} = \frac{T_{in} - T_a}{1/(1/(1/h_{r,in}) + 1/(1/h_{in}))} = (T_{in} - T_a)(h_{r,in} + h_{in})$$

That is to say, the temperature of the steel frame is:

$$T_a = T_{in} - \frac{T_{in} - T_{out}}{R(h_{r,in} + h_{in})} = 20^\circ \text{ C} - \frac{20^\circ \text{ C} - 0^\circ \text{ C}}{0.38 \text{ m}^2 \text{ K/W} \times (0.51 \text{ W/m}^2 \text{ K} + 3 \text{ W/m}^2 \text{ K})} = 5^\circ \text{ C}$$

From psychrometric chart, the dew point temperature of the indoor air (20°C , $\text{RH}=45\%$) is around 7.9°C , which is higher than the steel frame inside surface temperature (5°C). That is to say, the water will condense on the inside surface of the steel frame.

Note: from our calculation, the radiation and convection's total R value equals $0.38m^2.K/W$, which is much bigger than the steel frame's conduction $R=0.0013m^2.K/W$.

(c). If the inside surface of the steel is painted black, the radiation heat transfer through the inside surface will increase, which will increase the total heat transfer through the steel frame. Then the temperature of the steel frame will increase, which will cause better condensation problems (reducing condensation). If we re-calculate the temperature of the steel frame using black inside surface, we can get the temperature of the steel frame equaling to $8.83^{\circ}C$, which is higher than the indoor air dew point temperature ($7.9^{\circ}C$).

Problem 2

First, we may need to calculate the parameters of state 3:

From psychometric chart, we can get:

State 1: enthalpy $h_1=81$ kJ/kg air, humidity ratio $w_1=17.8$ g/kg air

State 2: enthalpy $h_2=56$ kJ/kg air, humidity ratio $w_2=12$ g/kg air

Mixing state 1 and state 2 air achieves state 3, we have equations:

$$m = m_1 + m_2 = 10kg / s$$

$$h_3 = \frac{m_1 h_1 + m_2 h_2}{m} = \frac{3 \times 81 + 7 \times 56}{10} = 63.5kJ / kg \cdot air$$

$$w_3 = \frac{m_1 w_1 + m_2 w_2}{m} = \frac{3 \times 17.8 + 7 \times 12}{10} = 13.74g / kg \cdot air$$

$$T_3 = \frac{m_1 T_1 + m_2 T_2}{m} = \frac{3 \times 35 + 7 \times 25}{10} = 28^{\circ} C$$

(a). From psychometric chart, we can get:

State 4: enthalpy $h_4=39.5$ kJ/kg air, humidity ratio $w_4=10$ g/kg air

The energy used to cool the air from state 3 to state 4 by the cooling heat exchanger A is:

$$Q_A = m(h_3 - h_4) = 10 \times (63.5 - 39.5) = 240kJ / s$$

The energy used to re-heating the air from state 4 to state 5 by the electrical heater B is:

$$Q_B = mCp(T_5 - T_4) = 10 \times 1.01 \times (18 - 14) = 40.4kJ / s$$

So the total energy consumed by the cooling heat exchanger A and B is 280.4 kJ/s.

(b). If the air conditioning system is running under steady state, the moisture generation in the room is:

$$m_v = m(w_2 - w_5) = m(w_2 - w_4) = 10 \times (12 - 10) = 20g / s$$

(c). Please see the attached psychometric chart.