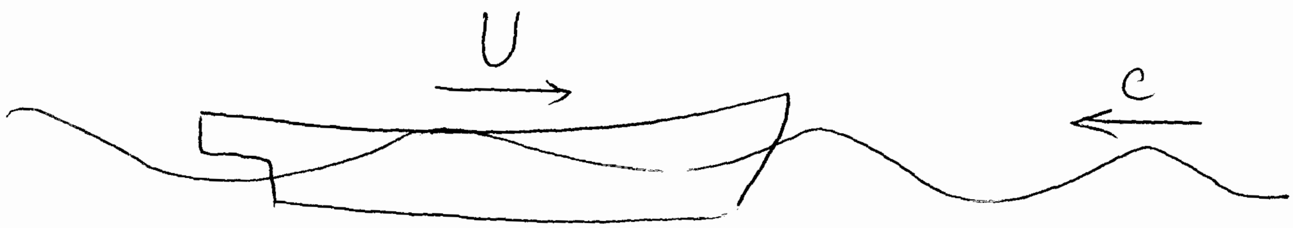
SPEED EFFECTS

SEAKEEPING

Speed Effects



When a vessel moves at speed U against the waves then it encounters the waves at a faster rate than when it is stationary. Now the relative speed of vessel and wave crest is $c+U$, so it takes τ time to run a wavelength λ:

$$\tau = \frac{\lambda}{U+c}$$

This means that there is an effective frequency $f_e = 1/\tau = U+c/\lambda$ and an effective circular frequency w_e

$$w_e = 2\pi f_e = \frac{2\pi}{\lambda}(U+c) = (U+c)k$$

Recall $c = \omega / k$, so

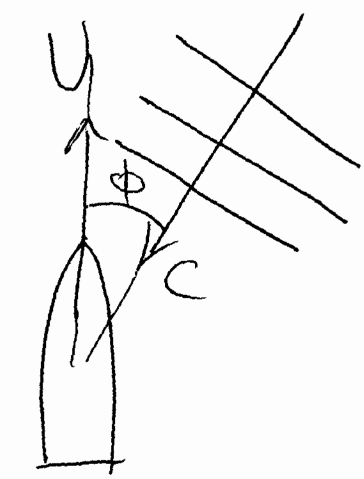
$$\omega_e = \omega + kU$$

For deep water waves $\omega^2 = kg$, hence

$$\boxed{\omega_e = \omega + \frac{\omega^2}{g} U}$$

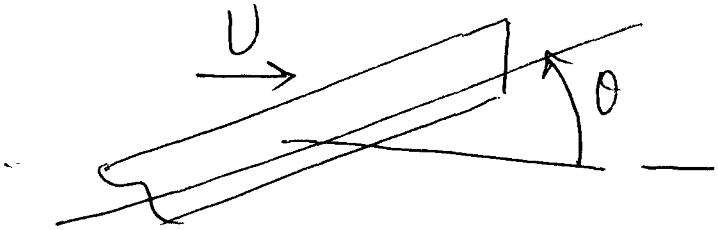
gives the <u>frequency of encounter</u> in <u>head</u> seas in deep water.

If the waves come at an angle ϕ with respect to the boat direction, then the same arguments give:



$$\boxed{\omega_e = \omega + \frac{\omega^2}{g} U \cos\phi}$$

In addition, when a ship has forward speed U against the waves, then the motions are further coupled.

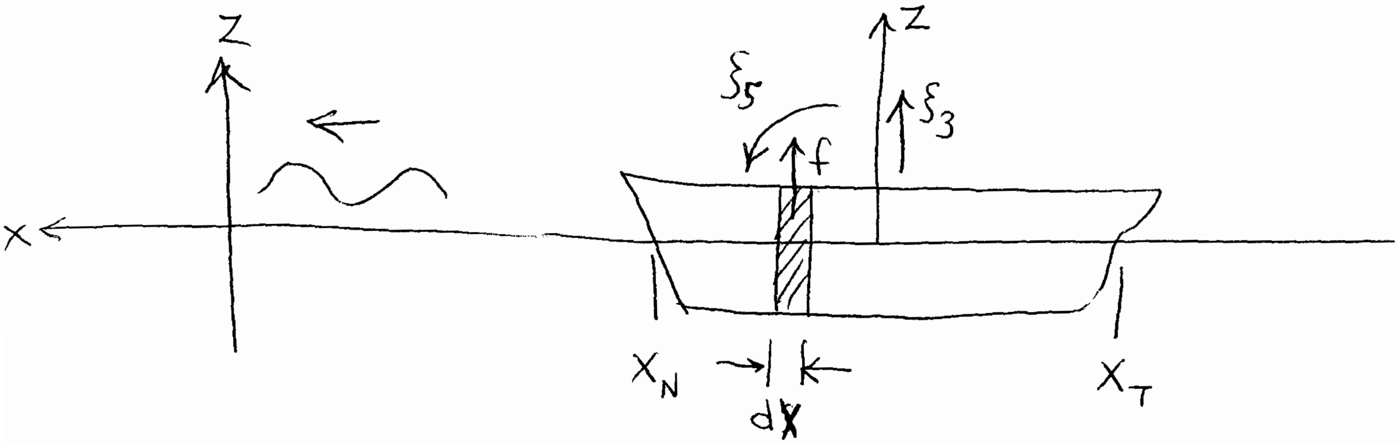


For example, if a ship pitches up by an angle θ

the forward velocity U makes it equivalent to having a heave velocity down by $U \sin\theta$ (the component of U in the instantaneous z-axis of the ship).

Hence, in addition to changing the frequency of encounter, speed causes cross-coupling of the hydrodynamic coefficients.

COUPLED HEAVE - PITCH MOTIONS



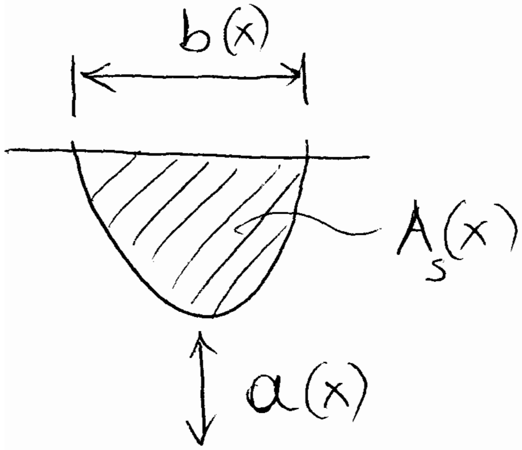
Excitation force and moment $(U=0)$

$f(x,t)$ is the force per unit length at x, t

$$F_3(t) = \int_{x_T}^{x_N} f(x,t)\,dx$$

$$F_5(t) = -\int_{x_T}^{x_N} x\, f(x,t)\,dx$$

For a sinusoidal wave of amplitude a and frequency ω we define by $a_{33}(x)$ the heave added mass of the section at x, $A_s(x)$ the cross sectional area at x and $b(x)$ the beam of the section



$$C_{33}(x) \simeq \rho g b(x)$$

$$f(x,t) = C_{33}\,\eta(x,t) + \left[\rho A(x) + a_{33}(x)\right] \frac{dw}{dt}\left(x, z=-\frac{T}{2}, t\right)$$

$$= \rho g b(x)\, a \cos(\omega t - kx) + \left[\rho A(x) + a_{33}(x)\right]\left(-a\omega^2\right) \cos(\omega t - kx)$$

Often, the following approximation gives quite good results

$$f(x,t) \simeq \rho g b(x)\, a \cos(\omega t - kx)$$

When
$$\boxed{F_3(t) = \rho g a \int_{x_T}^{x_N} b(x) \cos(\omega t - kx)\, dx}$$

$$\boxed{F_5(t) = -\rho g a \int_{x_T}^{x_N} x\, b(x) \cos(\omega t - kx)\, dx}$$

For example, for a rectangular-shaped ship $b(x) = B$, $x_N = \frac{L}{2}$, $x_T = -\frac{L}{2}$

$$F_3(t) = \rho g a \frac{B}{\cancel{k}} 2 \cos\omega t \sin\left(\frac{kL}{2}\right)$$

$$F_5(t) = -\rho g a \frac{B}{k^2} \left\{ kL \cos\left(\frac{kL}{2}\right) - 2\sin\left(\frac{kL}{2}\right) \right\} \sin\omega t$$

when $\omega \to 0 \Rightarrow k \to 0$

$$\Rightarrow \cos\left(\frac{kL}{2}\right) \sim 1 - \frac{1}{2}\left(\frac{kL}{2}\right)^2, \quad \sin\left(\frac{kL}{2}\right) \sim \frac{kL}{2} - \frac{1}{6}\left(\frac{kL}{2}\right)^3$$

$$\Rightarrow \boxed{\begin{aligned} F_3(t) &\simeq (\rho g BL)\, a \cos\omega t \\ &= C_{33}\, \eta(x=0,t) \end{aligned}}$$

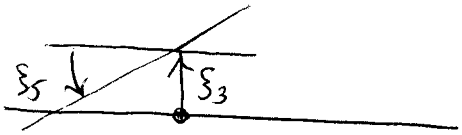
$C_{33} = \rho g (BL)$

$$\boxed{\begin{aligned} F_5(t) &\simeq +(\rho g a k)\left(\frac{BL^3}{12}\right) \sin\omega t \\ &= C_{55} \frac{d\eta}{dx}(x=0,t) \end{aligned}}$$

$C_{55} = \rho g \frac{BL^3}{12}$

<u>Note</u>: the expansion for $F_5(t)$ needs special care since third order terms are needed

Added Mass



Acceleration at x

$$\frac{d^2\xi}{dt^2} = \frac{d^2\xi_3}{dt^2} - x\,\frac{d^2\xi_5}{dt^2}$$

$\Rightarrow$ added mass term at x with 2-D added mass $a_{33}(x)$ is

$$-\,a_{33}(x)\,\frac{d^2\xi}{dt^2}$$

$$F_{R3}(t) = -\int_{x_T}^{x_N} a_{33}(x)\,\ddot{\xi}\,dx$$

$$= -A_{33}\,\ddot{\xi}_3 - A_{35}\,\ddot{\xi}_5$$

$$F_{R5}(t) = +\int_{x_T}^{x_N} x\,a_{33}(x)\,\ddot{\xi}\,dx$$

$$= -A_{53}\,\ddot{\xi}_3 - A_{55}\,\ddot{\xi}_5$$

$$A_{33} = \int_{x_T}^{x_N} a_{33}(x)\,dx$$

$$A_{35} = A_{53} = -\int_{x_T}^{x_N} x\,a_{33}(x)\,dx$$

$$A_{55} = \int_{x_T}^{x_N} x^2\,a_{33}(x)\,dx$$

Governing equations

$$\left.\begin{aligned}
&m\ddot{\xi}_3 - m x_G \ddot{\xi}_5 + C_{33}\xi_3 + C_{35}\xi_5 \\
&\qquad = F_3(t) - A_{33}\ddot{\xi}_3 - A_{35}\ddot{\xi}_5 \\
&I_{yy}\ddot{\xi}_5 - m x_G \ddot{\xi}_3 + C_{53}\xi_3 + C_{55}\xi_5 \\
&\qquad = F_5(t) - A_{53}\ddot{\xi}_3 - A_{55}\ddot{\xi}_5
\end{aligned}\right\}$$

EFFECTS OF FORWARD SPEED

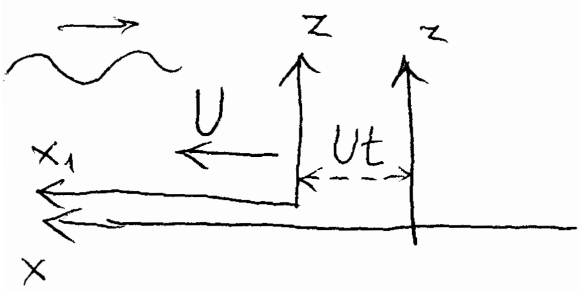
The first effect is that the excitation forces are at frequency ω_e. Indeed the elevation (head seas)

$$\eta(x,t) = a\cos(\omega t + kx)$$

in a moving frame, $x_1 = x - Ut$, is seen as

$$\eta(x_1,t) = a\cos(\omega t + k(x_1 - Ut))$$

$$= a\cos([\omega + kU]t + kx_1)$$

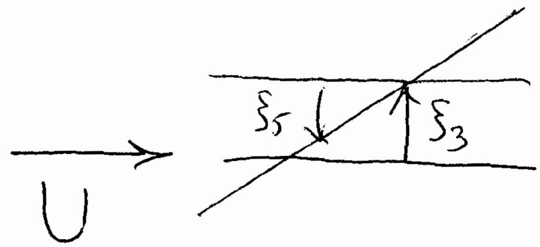


When the approximation on page (e-5) is used, called the Froude-Krylov approximation, then the expressions for $F_3(t)$, $F_5(t)$ do not change except for replacing ω by ω_e

$$F_3(t) = \rho g a \int_{x_T}^{x_N} b(x)\cos(\omega_e t - kx)\,dx$$

$$F_5(t) = -\rho g a \int_{x_T}^{x_N} x\, b(x)\cos(\omega_e t - kx)\,dx$$

The added mass forces, though, change because



the relative velocity between a ship-section at x and the fluid in the normal direction to the ship axis is

$$w \cong \frac{d\xi_3}{dt} - x\frac{d\xi_5}{dt} + U\xi_5$$

Now the added mass force is following the fluid particles

$$dZ = -\frac{d_F}{dt}\left(a_{33}(x)\, w(x,t)\right)$$

$$= -\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\left(a_{33}(x)\, w(x,t)\right)$$

$$\Rightarrow \boxed{F_{R3}(t) = \int_{x_T}^{x_N} dZ = -A_{33}\ddot{\xi}_3 - A_{35}\ddot{\xi}_5 - UA_{33}\dot{\xi}_5}$$

$$F_{R5}(t) = -\int_{x_T}^{x_N} x\, dZ = -A_{53}\ddot{\xi}_3 - A_{55}\ddot{\xi}_5 - UA_{53}\dot{\xi}_5 - U\int_{x_T}^{x_N} x\,\frac{\partial}{\partial x}\left[a_{33}(x)\, w(x,t)\right]dx$$

where
$$\int_{x_T}^{x_N} x \frac{\partial}{\partial x}\left[a_{33}(x)\, w(x,t)\right] dx$$

$$= -\int_{x_T}^{x_N} a_{33}(x)\, w(x,t)\, dx = -A_{33}\frac{d\xi_3}{dt} - A_{35}\frac{d\xi_5}{dt} - A_{33} U \xi_5$$

so finally

$$\boxed{F_{R5}(t) = -A_{53}\ddot{\xi}_3 - A_{55}\ddot{\xi}_5 - \cancel{A_{53} U \dot{\xi}_5} - U A_{33}\frac{d\xi_3}{dt} + U \cancel{A_{35}\frac{d\xi_5}{dt}} + U^2 A_{33}\xi_5}$$

These expressions ommit certain terms related to lift-like forces on the vessel.

Now the coupled equations of motion can be written as follows:

$$(m+A_{33})\ddot{\xi}_3 + (-mx_G + A_{35})\ddot{\xi}_5 + UA_{33}\dot{\xi}_5 + C_{33}\xi_3 + C_{35}\xi_5$$
$$= F_3(t)$$

$$(I_{yy} + A_{55})\ddot{\xi}_5 + (-mx_G + A_{53})\ddot{\xi}_3 + UA_{33}\dot{\xi}_3 +$$
$$+ (C_{55} - U^2 A_{33})\xi_5 + C_{53}\xi_3 = F_5(t)$$

The solution is found by setting

$$\xi_3 = a_3 \cos(\omega_e t + \psi_3)$$
$$\xi_5 = a_5 \cos(\omega_e t + \psi_5)$$

and solving for the unknown $\xi_3, \xi_5, \psi_3, \psi_5$