

22.01 Problem Set 1 Homework Solutions

(* please note that any number in brackets, (1.23), for example, refers to equation found in the text book)

Problems in Turner:

2.4 (1 points) The mass of one atom of aluminum is found by taking the grams per mole and using chemical conversions to calculate the mass of a single atom:

$$1 \text{ atom} \cdot \frac{26.98 \text{ g}}{\text{mol}} \cdot \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ atoms}} = 4.48 \times 10^{-23} \text{ g} = 4.48 \times 10^{-26} \text{ kg}$$

2.14 (1 points) The ionization potential is defined as the energy required to remove the electron from the ground state, the $n=1$ level. Using 2.12 and plugging in for Li^{+2} :

$$E = \frac{13.6Z^2}{n^2} = \frac{13.6(3)^2}{1^2} = 122.4 \text{ eV}$$

2.56 (1 points) Given:

$$\begin{aligned} E_K &= 8979 \text{ eV} \\ E_L &= 951 \text{ eV} \\ E_M &= 74 \text{ eV} \end{aligned}$$

By definition K_α and K_β are:

$$\begin{aligned} K_\alpha &= E_L - E_K = -8028 \text{ eV} \\ K_\beta &= E_M - E_K = -8905 \text{ eV} \end{aligned}$$

3.3 (3 points) The alpha particle would need to have enough energy to overcome the repulsive Coulomb barrier in a head on collision with ^{238}U . To calculate the Coulomb potential energy, we need to know the total distance between the centers of the two nuclei so they are "just touching." Using equation 2.2 and $A=4$ for the alpha atom and $A=238$ for the uranium atom:

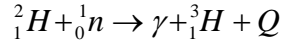
$$\begin{aligned} r_\alpha &= 1.3 \times 10^{-15} \cdot 4^{1/3} = 2.06 \times 10^{-15} \text{ m} \\ r_U &= 1.3 \times 10^{-15} \cdot 238^{1/3} = 8.06 \times 10^{-15} \text{ m} \end{aligned}$$

Therefore the total distance between the two charges is $r_\alpha + r_U = 1.02 \times 10^{-14} \text{ m}$. The unit charge for the alpha is $4e$ and for uranium, $92e$, where e is the unit charge $e = 1.6 \times 10^{-19} \text{ C}$. Solving for the Coulomb potential energy:

$$E_C = \frac{kZ_1Z_2}{r} = \frac{8.99 \times 10^9 \cdot 92 \cdot 2 \cdot (1.6 \times 10^{-19})^2}{1.02 \times 10^{-14}} = 4.15 \times 10^{-12} \text{ J} = 25.95 \text{ MeV}$$

A similar problem as this one is also in the text on page 56.

3.4 (2 points) The nuclear reaction is:

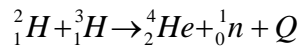


The energy, Q , released in the decay is the difference in the masses between the parent nucleus and the daughter nuclei. Therefore, this mass difference can be expressed using the mass excesses, and can be written:

$$\begin{aligned} Q &= \Delta_{\text{parent}} - \Delta_{\text{daughter}} \\ &= \Delta_{{}_2\text{H}} + \Delta_n - \Delta_{{}_3\text{H}} \\ &= 13.1359 + 8.0714 - 14.95 \\ &= 6.2573\text{Mev} \end{aligned}$$

where mass excesses were obtained from Appendix D. This energy is the energy of the photon that is emitted.

1. (2 points) The nuclear reaction is:



The energy, Q , released in the decay is the difference in the masses between the parent nucleus and the daughter nuclei. Therefore, this mass difference can be expressed using the mass excesses, and can be written:

$$\begin{aligned} Q &= \Delta_{\text{parent}} - \Delta_{\text{daughter}} \\ &= \Delta_{{}_2\text{H}} + \Delta_{{}_3\text{H}} - \Delta_n - \Delta_{{}_4\text{He}} \\ &= 13.1359 + 14.95 - 8.0714 - 2.4248 \\ &= 17.59\text{Mev} \end{aligned}$$

2). (3 points)

a). When the alpha particle nears the gold nucleus, it will feel the Coulomb force. The Coulomb force, and energy, is dependent on a distance between two charged particles. Therefore the minimum distance that the alpha can approach the gold nucleus will be at which distance r the energy of the alpha particle equals the Coulomb potential of the gold nucleus:

$$E_\alpha = \frac{kZ_1Z_2}{r} = \frac{8.99 \times 10^9 \cdot 79 \cdot 2 \cdot (1.6 \times 10^{-19})^2}{r} = 1.2304 \times 10^{-12} \text{ J} = 7.69 \text{ MeV}$$

which gives an $r = 2.955 \times 10^{-14} \text{ m}$.

b). This is the same type of question as 3.3 but for gold.

Using equation 2.2 and $A=4$ for the alpha atom and $A=238$ for the uranium atom:

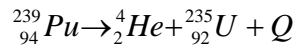
$$r_{\alpha} = 1.3 \times 10^{-15} \cdot 4^{1/3} = 2.06 \times 10^{-15} \text{ m}$$

$$r_{Au} = 1.3 \times 10^{-15} \cdot 197^{1/3} = 7.56 \times 10^{-15} \text{ m}$$

Therefore the total distance between the two charges is $r_{\alpha} + r_U = 9.624 \times 10^{-15}$ m. The unit charge for the alpha is $4e$ and for gold, $79e$, where e is the unit charge $e = 1.6 \times 10^{-19}$ C. Solving for the Coulomb potential energy:

$$E_C = \frac{kZ_1Z_2}{r} = \frac{8.99 \times 10^9 \cdot 79 \cdot 2 \cdot (1.6 \times 10^{-19})^2}{9.624 \times 10^{-15}} = 3.778 \times 10^{-12} \text{ J} = 23.61 \text{ MeV}$$

3). (4 points) The nuclear decay equation is:



a). The energy, Q , released in the decay is the difference in the masses between the parent nucleus and the daughter nuclei. Therefore, this mass difference can be expressed using the mass excesses, and can be written:

$$\begin{aligned} Q &= \Delta_{\text{parent}} - \Delta_{\text{daughter}} - \Delta_{\alpha} & (3.13) \\ &= \Delta_{\text{Pu}} - \Delta_{\text{U}} - \Delta_{\alpha} \\ &= 48.6 - 40.93 - 2.4248 \\ &= 5.2452 \text{ MeV} \end{aligned}$$

(* Note, this energy is shared between the alpha particle and the daughter nucleus)

b). The energy of the alpha particle is determined by using the conservation of momentum and of energy. Section 3.3 in the text has a good description and proof of the following formula:

$$E_{\alpha} = \frac{MQ}{m + M} \quad (3.18)$$

where M is the mass of the recoil daughter, and m is the mass of the alpha particle.

Therefore, for the Radon decay:

$$E_{\alpha} = \frac{(235)(5.2452)}{4 + 235} = 5.157 \text{ MeV}$$

Notice that most of the energy is given to the daughter nucleus.

Using conservation of energy, the recoil energy of uranium is:

$$E_U = Q - E_{\alpha} = 5.2452 - 5.157 = 0.088 \text{ MeV}$$

c). The mass of 1 alpha particle is:

$$1 \text{ atom} \cdot \frac{4.002 \text{ g}}{\text{mol}} \cdot \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ atoms}} = 6.65 \times 10^{-24} \text{ g} = 6.65 \times 10^{-27} \text{ kg}$$

The velocity of the alpha particle is:

$$E = \frac{1}{2}mv^2$$

$$5.157 \text{ MeV} = 8.25 \times 10^{-13} \text{ J} = \frac{1}{2}(6.65 \times 10^{-27})v^2$$

which gives $v = 1.58 \times 10^7 \text{ m/s}$. (We can justify not using relativity because this is .05 the speed of light)

4). (6 points)

a). First, we need to calculate the mass excess of ^{56}Fe . We are given $M = 55.934942 \text{ amu}$ and $A=56$. Also $1 \text{ amu} = 931.5 \text{ MeV}$, so:

$$\Delta_{^{56}\text{Fe}} = M - A = 55.934942 - 56 = -.065058 \text{ amu} = -60.602 \text{ MeV}$$

The nuclear equation to remove a neutron and the calculation of Q, the energy needed to remove the neutron:

$$^{56}_{26}\text{Fe} \rightarrow ^1_0\text{n} + ^{55}_{26}\text{Fe} + Q$$

$$Q = \Delta_{\text{parent}} - \Delta_{\text{daughter}}$$

$$= \Delta_{^{56}\text{Fe}} - \Delta_{\text{n}} - \Delta_{^{55}\text{Fe}}$$

$$= -60.602 - (-57.474) - 8.0714$$

$$= -11.2 \text{ MeV}$$

As expected, you need to supply energy to remove the neutron.

b) The nuclear equation to remove a proton and the calculation of Q, the energy needed to remove the proton:

$$^{56}_{26}\text{Fe} \rightarrow ^1_1\text{p} + ^{55}_{25}\text{Mn} + Q$$

$$\begin{aligned}
 Q &= \Delta_{parent} - \Delta_{daughter} \\
 &= \Delta_{^{56}\text{Fe}} - \Delta_p - \Delta_{^{55}\text{Mn}} \\
 &= -60.602 - (-57.705) - 7.2890 \\
 &= -10.19\text{MeV}
 \end{aligned}$$

Again, as expected, you need to supply energy to remove a proton.

c). In order to find the energy to completely dismantle ^{56}Fe into individual components, we need to calculate the total binding energy of the atom, since this is the energy to break it apart into each part.

Working in terms of amu, and then using the conversion factor (1 amu = 931.49 MeV):

$$\begin{aligned}
 1 \text{ proton} &= 1.0073 \text{ amu} \\
 1 \text{ neutron} &= 1.0087 \text{ amu} \\
 1 \text{ electron} &= .00055 \text{ amu}
 \end{aligned}$$

First we sum up all of the energy from the total individual nucleons and electrons in ^{56}Fe :

$$\begin{aligned}
 &\Rightarrow (\text{number of protons} \times \text{mass of protons}) + (\text{number of neutrons} \times \text{mass of neutrons}) + (\text{number of electrons} \times \text{mass of electrons}) \\
 &\Rightarrow 26(1.0073) + 30(1.0087) + 26(.00055) \quad (* \text{ Notice that the electron mass is so small that it could just be neglected}) \\
 &\quad = 56.4651 \text{ amu}
 \end{aligned}$$

We know that some mass will be converted to energy. We can use mass excess, Δ , to calculate the mass of the Fe atom:

$$\Delta = M - A$$

From above $\Delta = -0.0650588$ amu, and $A = 56$ amu:

$$\begin{aligned}
 \Delta &= -0.0650588 = M - 56 \\
 M &= 55.9349 \text{ amu}
 \end{aligned}$$

The difference in the mass between the total sum of nucleons and the actual mass of the atom gives the binding energy:

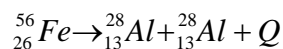
$$\text{B.E.} = 56.4651 - 55.9349 = .5302 \text{ amu or } 493.84 \text{ MeV}$$

The energy to dismantle the nucleus is therefore approximately 494 MeV.

d). First, we need to calculate the mass excess of ^{28}Al . We are given $M = 27.981910$ amu and $A = 28$.

$$\Delta_{^{28}\text{Al}} = M - A = 27.981910 - 28 = -0.01809 \text{ amu} = -16.851 \text{ MeV}$$

The nuclear equation to remove a neutron and the calculation of Q , the energy needed to remove the neutron:



$$\begin{aligned}
 Q &= \Delta_{parent} - \Delta_{daughter} \\
 &= \Delta_{^{56}\text{Fe}} - 2 \cdot \Delta_{^{28}\text{Al}} \\
 &= -60.602 - 2(-16.851) \\
 &= -26.9\text{MeV}
 \end{aligned}$$

5) (2 points)

a). The energy needed to excite an electron from the ground level, $n=1$, to the first excited level, $n=2$, in hydrogen is:

$$E = -13.6Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = -13.6 \cdot 1^2 \left(\frac{1}{2^2} - \frac{1}{1^2} \right) = 10.2\text{eV} \quad (2.15)$$

b). The orbital velocity of the $n=2$ level in hydrogen is:

$$v_n = \frac{k_0 Z e^2}{n \hbar} \quad (2.9)$$

$$v_2 = \frac{8.99 \times 10^9 \cdot 1 \cdot (1.6 \times 10^{-19})^2}{2 \cdot 1.05 \times 10^{-34}} = 1.096 \times 10^6 \frac{\text{m}}{\text{s}}$$

6) (5 points) a). Using the Bohr theory of the hydrogen atom, and extending it to titanium, the energy of the electron in its n th shell is:

$$E = \frac{-13.6Z^2}{n^2} \text{ eV} \quad (2.12)$$

where Z is the number of protons in the atom. Calculating now the E energy for levels:

K shell: $n=1$
 L shell: $n=2$
 M shell: $n=3$
 $Z=22$ (titanium)

Gives:

$$\begin{aligned}
 E_K &= -6.582 \text{ keV} \quad (-1.05 \times 10^{-15} \text{ J}) \\
 E_L &= -1.6455 \text{ keV} \quad (-2.633 \times 10^{-16} \text{ J}) \\
 E_M &= -0.7313 \text{ keV} \quad (-1.17 \times 10^{-16} \text{ J})
 \end{aligned}$$

b). By definition K_α and K_β are:

$$\begin{aligned}
 K_\alpha &= E_L - E_K = 4.9365 \text{ keV} \quad (7.867 \times 10^{-16} \text{ J}) \\
 K_\beta &= E_M - E_K = 5.8507 \text{ keV} \quad (9.36 \times 10^{-16} \text{ J})
 \end{aligned}$$

c). The energy of the emitted photon is proportional to the wavelength by:

$$E = \frac{hc}{\lambda} \quad (2.13)$$

where $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$, $c = 3 \times 10^8 \text{ m/s}$, and getting λ in meters gives:

$$\lambda_{\alpha} = 2.53 \times 10^{-10} \text{ m}$$

$$\lambda_{\beta} = 2.12 \times 10^{-10} \text{ m}$$