SHORT-TERM

&

LONG-TERM

STATISTICS

031505

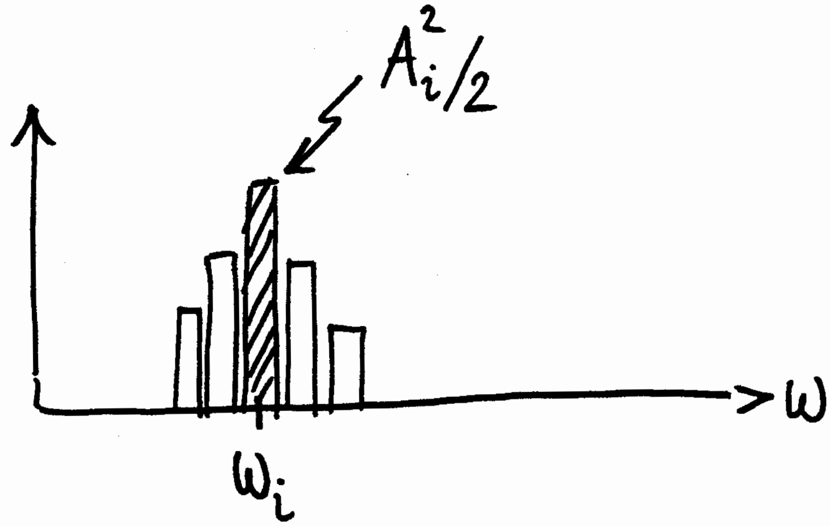
$$y(t,\mathfrak{J}) = \sum_{i=1}^{N} A_i \cos\left(\omega_i t + \phi_i(\mathfrak{J})\right)$$

$$E\{y(t,\mathfrak{J})\} = 0$$

$$E\{y^2(t,\mathfrak{J})\} = \sum_{i=1}^{N} \frac{A_i^2}{2}$$

$$R(\tau) = E\{y(t,\mathfrak{J})\, y(t+\tau,\mathfrak{J})\} =$$

$$= \sum_{i=1}^{N} \frac{A_i^2}{2} \cos(\omega_i \tau)$$



SPECTRUM

$S_y(\omega)$

Fourier Transform of $R(\tau)$

$$u(t) \longrightarrow \boxed{H(\omega)} \longrightarrow y(t)$$

$$u(t) = \sum_{i=1}^{N} u_i \cos\left(\omega_i t + \phi_i(\zeta)\right)$$

$$R_u(\tau) = \sum_{i=1}^{N} \frac{u_i^2}{2} \cos(\omega_i \tau)$$

$$S_u(\omega)$$

$$y(t) = \sum_{i=1}^{N} \underbrace{u_i \left|H(\omega_i)\right|}_{y_i} \cos\left(\omega_i t + \phi_i(\zeta) + \measuredangle H(\omega_i)\right)$$

$$R_y(\tau) = \sum_{i=1}^{N} \frac{u_i^2}{2} \left|H(\omega_i)\right|^2 \cos(\omega_i \tau)$$

$$\boxed{S_y(\omega) = S_u(\omega) \left|H(\omega)\right|^2}$$

spectrum of sea $S_u(\omega)$

transfer function (HYDRO) $H(\omega)$

spectrum of response $S_y(\omega)$

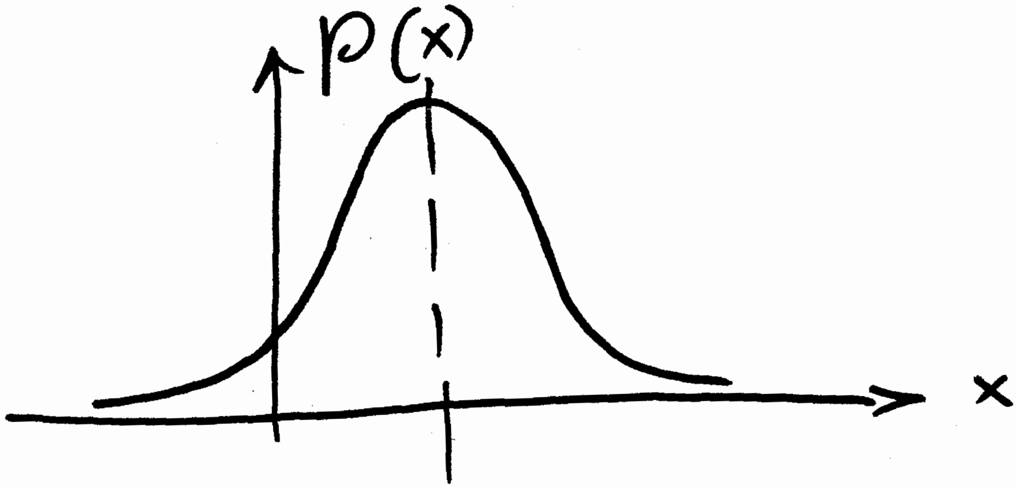
CENTRAL LIMIT THEOREM

$$x = x_1 + x_2 + \cdots + x_n$$

$x_1, x_2 \ldots x_n$ random variables

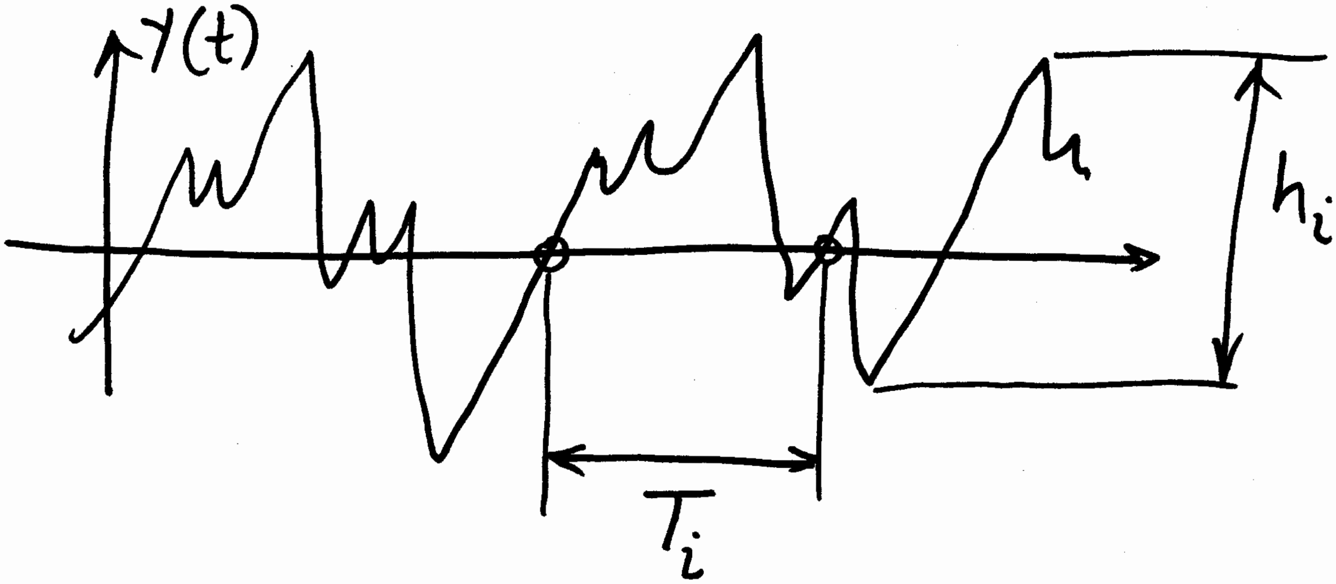
x becomes a Gaussian r.v. as

$n \to \infty$



SHORT TERM STATISTICS

Short-term ~ 1 hour



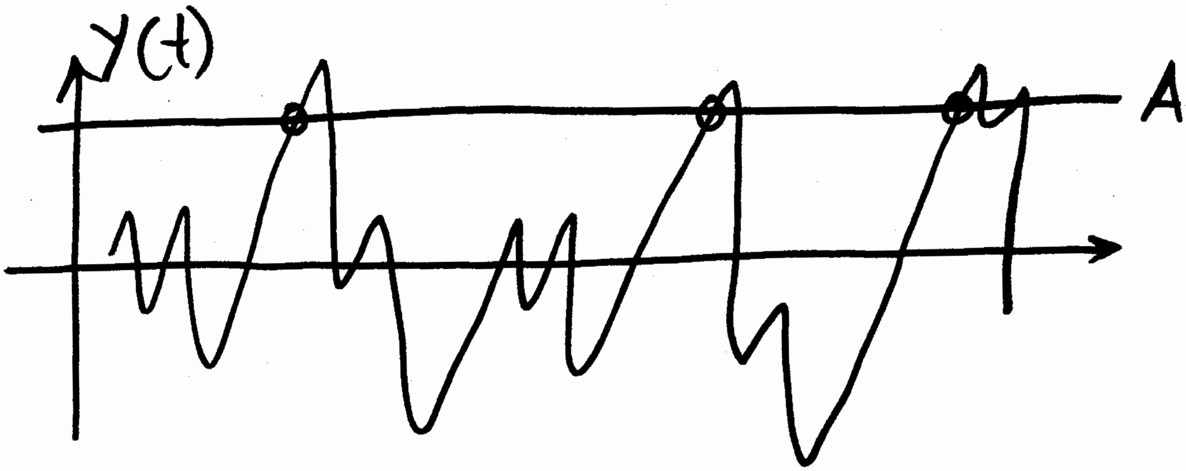
Given $S_y(\omega)$, $\varepsilon^2 = 1 - \frac{M_2^2}{M_0 M_4}$

$$M_0 = \int_0^\infty S_y(\omega)\, d\omega$$

$$M_2 = \int_0^\infty S_y(\omega)\, \omega^2 d\omega$$

$$M_4 = \int_0^\infty S_y(\omega)\, \omega^4 d\omega$$

How often $y(t)$ exceeds a level A



$\bar{n}(A)$ = average number of times per second above A (upcrossing)

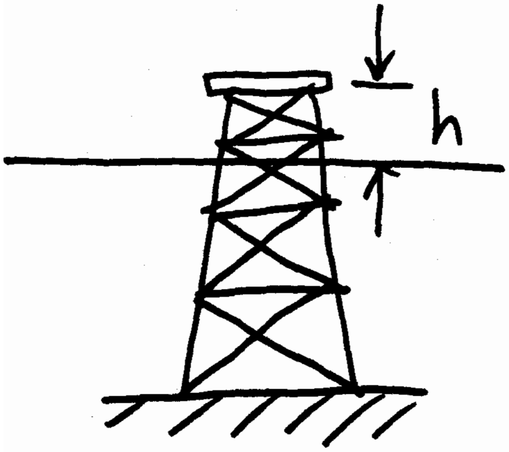
$$\bar{n}(A) = \frac{1}{2\pi}\sqrt{\frac{M_2}{M_0}}\, e^{-\frac{A^2}{2M_0}}$$

$$\Rightarrow \bar{n}(0) = \frac{1}{\bar{T}} = \frac{1}{2\pi}\sqrt{\frac{M_2}{M_0}}$$

$$\Rightarrow \bar{n}(A) = \frac{1}{\bar{T}}\, e^{-A^2/2M_0}$$

EXAMPLE

Platform in a storm with variance $\sigma = 2^m$, period $\bar{T} = 8^s$. Design h to have on average flooded deck once every 10 min



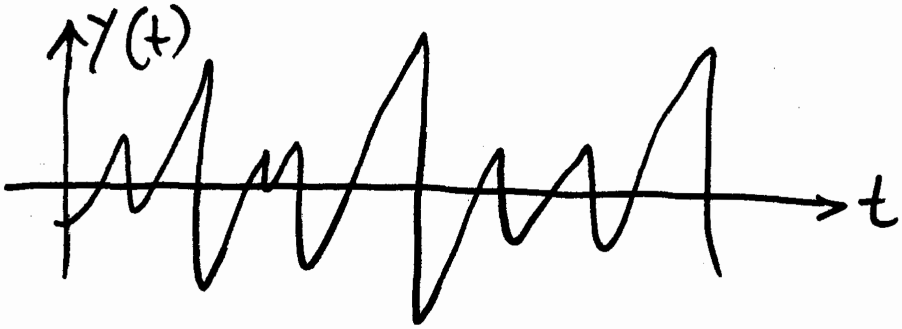
$$M_0 = \sigma^2 \ , \ \bar{T} = 8^s$$

$$\bar{n}(h) = \frac{1}{\bar{T}} e^{-h^2/2M_0} = \frac{1}{8^s} e^{-h^2/(2*2^2)}$$

$$= \frac{1}{10^{min}} = \frac{1}{10^{min}\, 60^{s}/_{min}}$$

$$\Rightarrow h = \sqrt{-2M_0 \ln\left(\frac{\bar{T}}{10*60}\right)} = \underline{\underline{5.87^m}}$$

ONE-THIRD HIGHEST AVERAGE HEIGHT



→ Measure 12 consecutive heights

6, 5, 3, 4, 7, 11, 8, 9, 5, 4, 2, 5

→ Keep one-third highest (four)

7, 11, 8, 9

→ find average 8.75 $\boxed{\overline{h^{1/3}} = 8.75}$

APPROXIMATELY (valid for narrow-band spectrum)

$$\boxed{\overline{h^{1/3}} \simeq 4\sqrt{M_0}}$$

Long-term statistics

Design for years of operation

TABLE OF STORMS

Storm 1	period $\overline{T}_1$	height $\overline{H^{1/3}_1}$	probability
2	$\overline{T}_2$	$\overline{H^{1/3}_2}$	
3			
...			

probability that any wave will have height larger than h_0

$$P(h > h_0) \cong E\left\{ \exp\left[-\frac{2h_0^2}{\left(H^{1/3}\right)^2} \right] \right\}$$

HUNDRED YEAR WAVE height h_{100}

$$p(h > h_{100}) = \frac{1}{(100\,\mathrm{yr}/\bar{T})} = \frac{\bar{T}}{3.15 \cdot 10^9}$$

WEIBULL DISTRIBUTION

Extreme events follow the Weibull distribution

$$p(x > x_0) = \exp\left\{-\left(\frac{x_0 - x_1}{x_2 - x_1}\right)^{\gamma}\right\}$$

x_1, x_2, γ parameters to be determined from storm data

$$\ln\left[-\ln\left[p(x > x_0)\right]\right] = \gamma\left\{\ln[x_0 - x_1] - \ln[x_2 - x_1]\right\}$$

Table 1a *Number of visual wave heights and periods at Weather Station India as given by Walden (1964)*

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Table 1b *Number of measured wave heights and periods at Weather Station India based on 2400 records as given by Draper & Squire (1967)*

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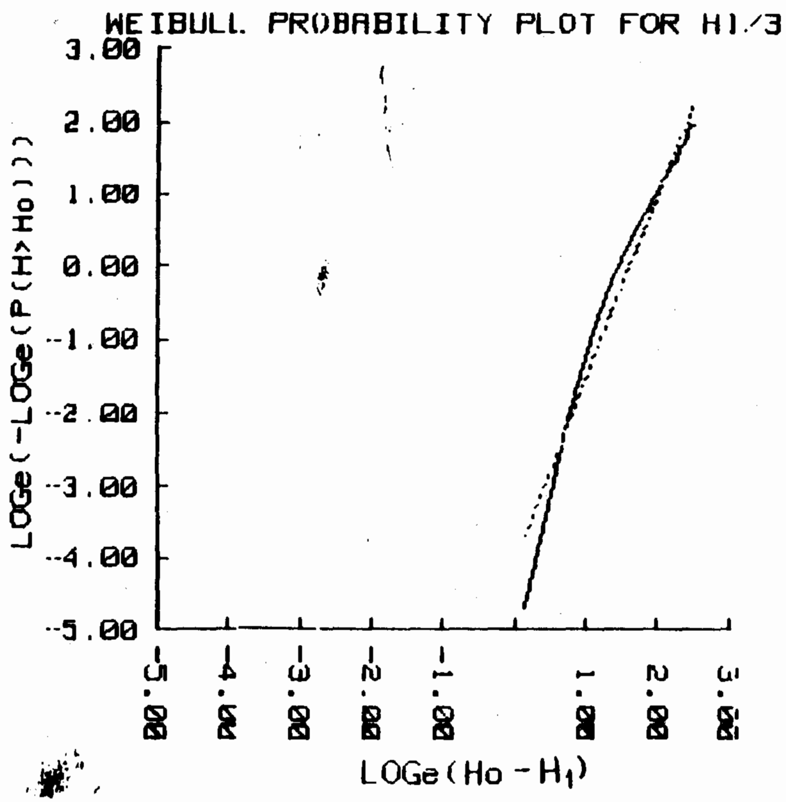
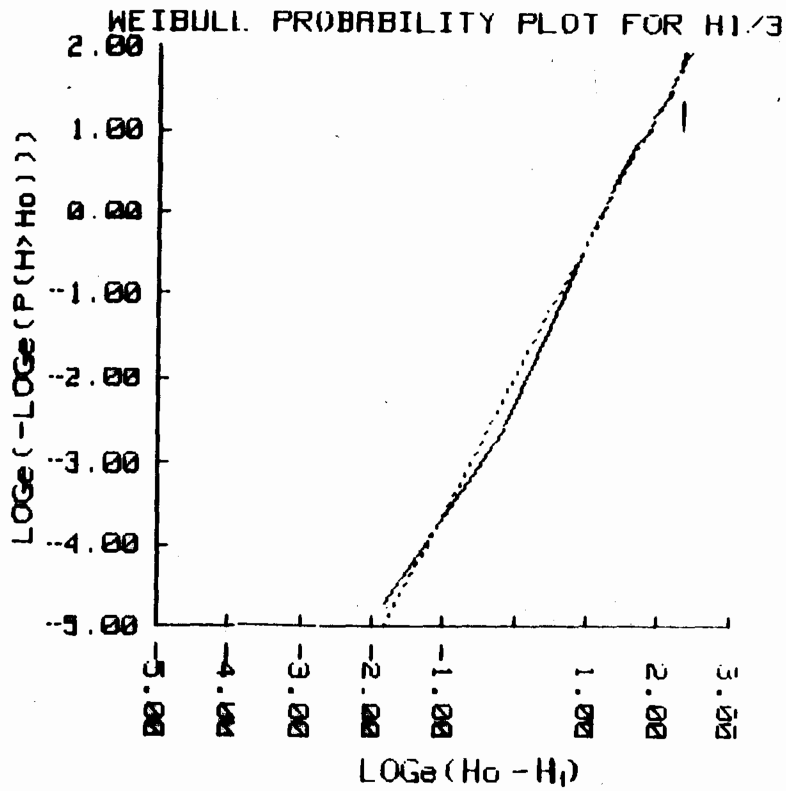


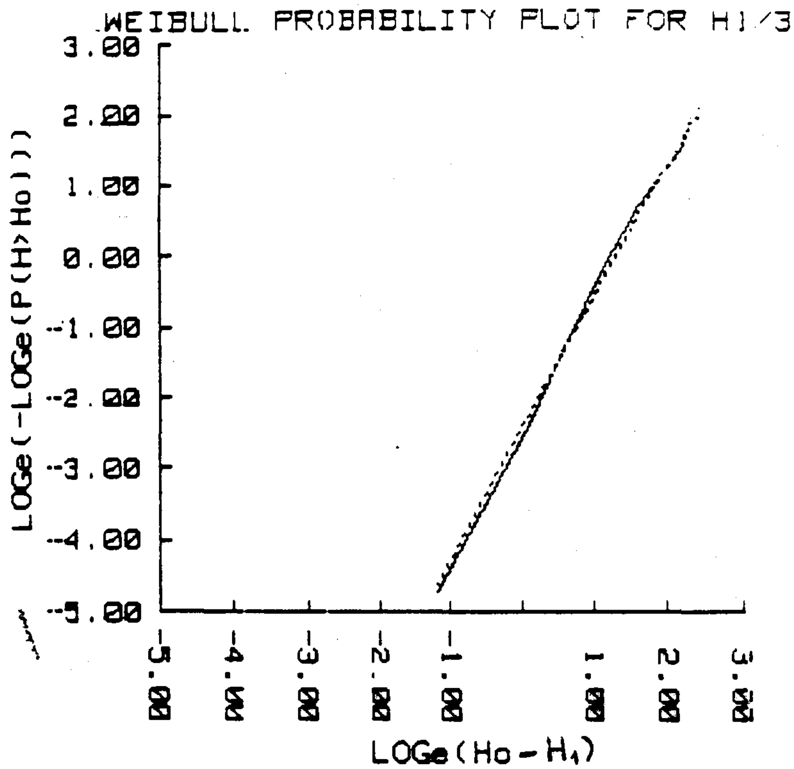
$H_2 = 5.02$
$H_1 = 0.$
$\gamma = 2.53$

Figure 1a (Data from Table 1b)



$H_2 = 4.38$
$H_1 = 1.$
$\gamma = 1.63$

Figure 1b



$H_2 = 4.59$

$H_1 = 0.85$

$\gamma = 1.85$

Figure 1c: Experimental data reported by Draper and Squire (1967) from Weather Station India (See Table 1)

$\bar{T} = 8.5$ sec

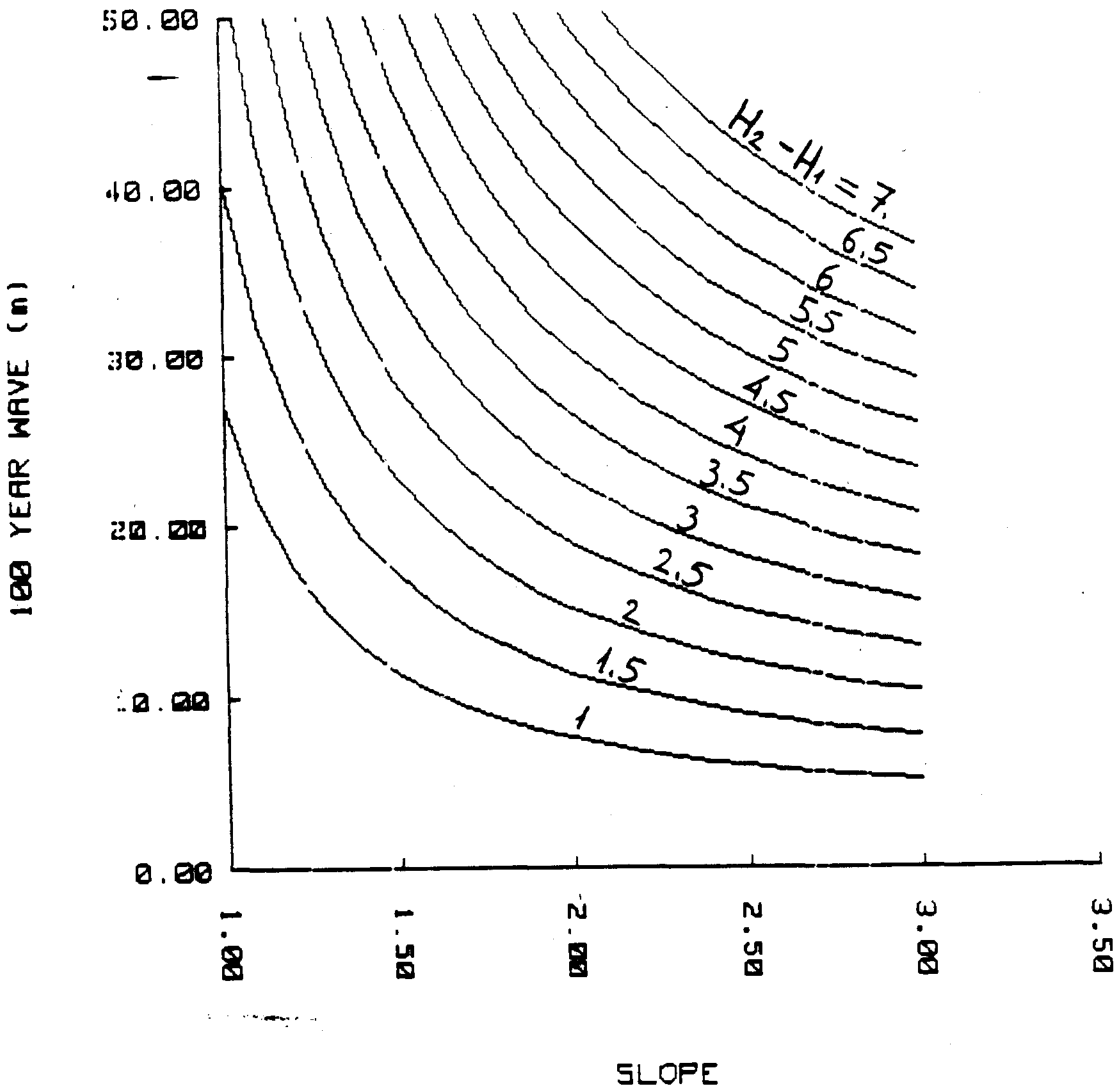


Fig. 2 The 100 year wave versus slope γ
Parameter $H_2 - H_1$
Average period 8.5 sec

$\bar{T} = 8.5$ sec.

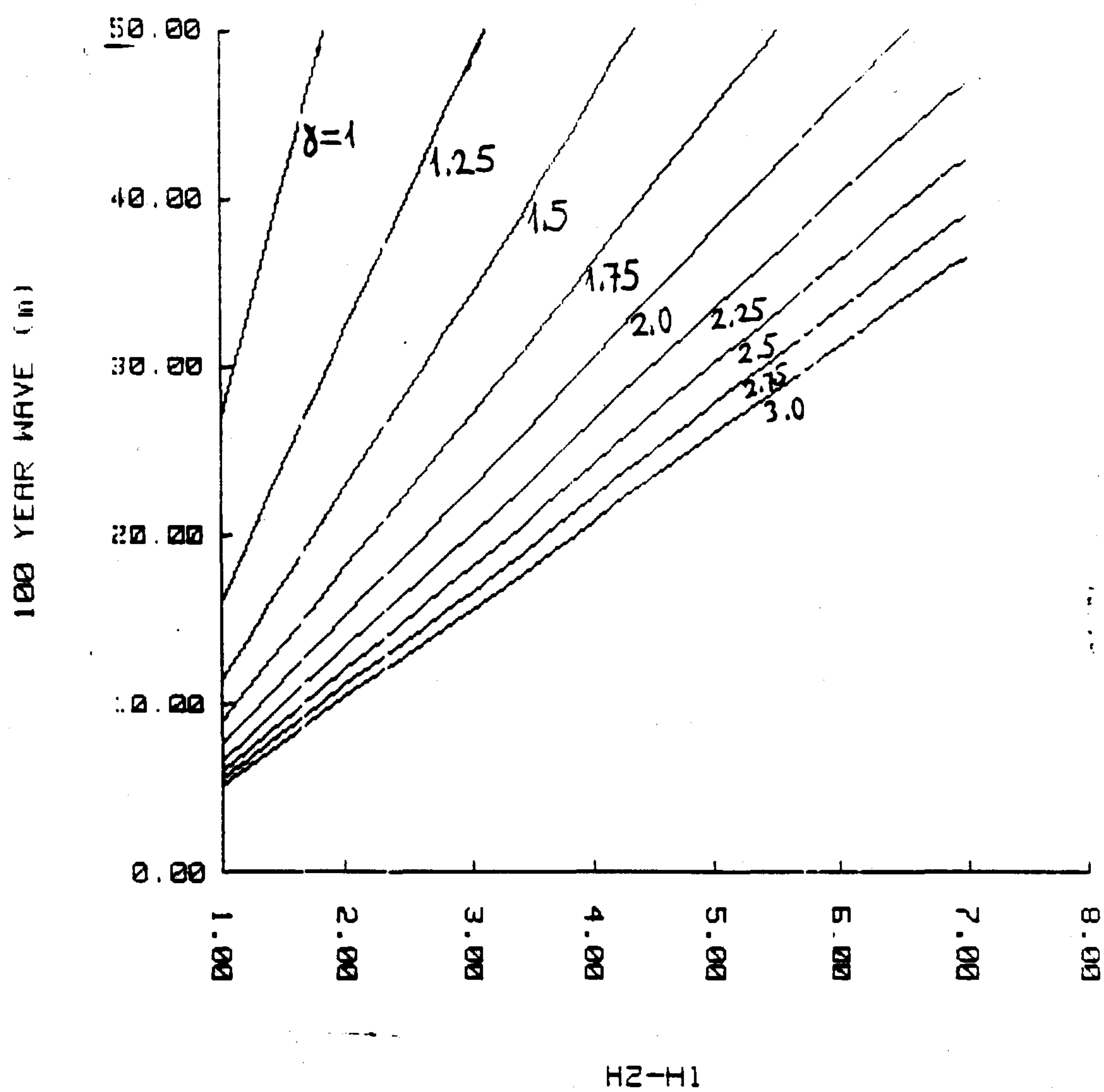


Fig. 3 The 100 year wave versus $H_2 - H_1$
Parameter γ
Average period 8.5 sec

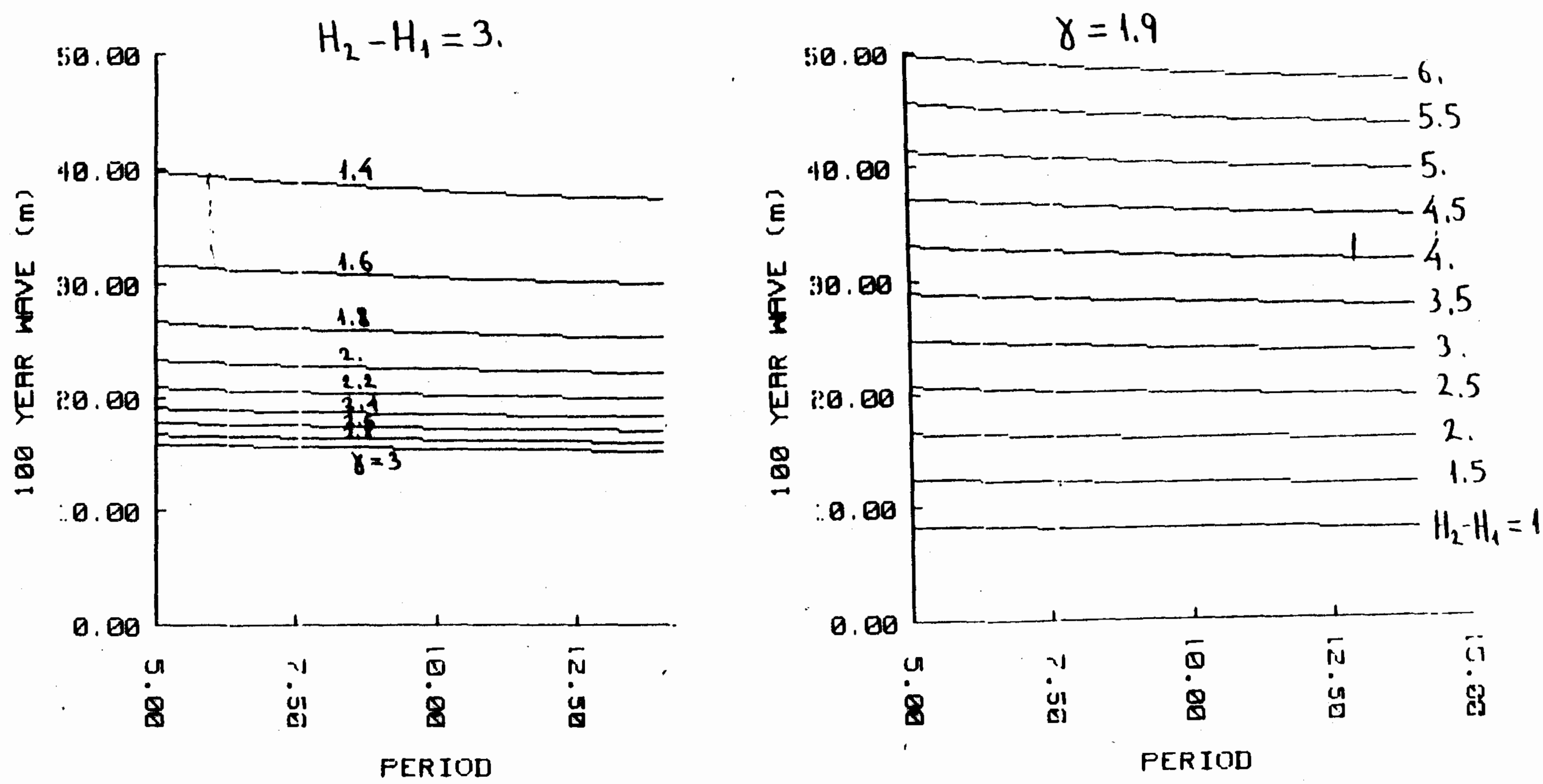


Fig: 4 ab : The influence of the average period $\overline{T}$ on the hundred year wave is small