

24.400□  
Proseminar in philosophy I

Fall 2003

A bit of “The Concept of Truth in Formalized Languages” in more modern notation

The language of the “calculus of classes” is a first-order language  $L$  whose logical vocabulary is ' $\forall$ ', ' $\exists$ ', ' $\sim$ ', an infinite supply of variables ' $x$ ', ' $x''$ ', ..., and parentheses '(', ')'. The sole non-logical item of vocabulary is the two place predicate ' $\subseteq$ '. The quantifiers of  $L$  are stipulated to range over classes of individuals and ' $\subseteq$ ' is interpreted as class inclusion.

A *formula* of the language of the calculus of classes is either a closed or open sentence. Thus ' $(\forall x'x' \subseteq x'')$ ' and ' $\sim(x'' \subseteq x')$ ' are formulas of the language.

DEFINITION 22

An infinite sequence of classes  $S$  *satisfies the formula*  $\phi$ , or  $\phi$  is *true relative to*  $S$ , iff:

either:

(a) There are numbers  $k$  and  $l$  such that  $\phi = \lceil v \subseteq \mu \rceil$ , where  $v$  is the  $k$ th variable (i.e. the variable whose superscript has  $k$  strokes) and  $\mu$  is the  $l$ th variable (i.e.  $\phi$  is the formula consisting of the  $k$ th variable, ' $\subseteq$ ', and the  $l$ th variable, in that order), and the  $k$ th member of  $S$  is a subset of the  $l$ th member of  $S$

or:

(b) There is a formula  $\psi$  such that  $\phi = \lceil \sim\psi \rceil$  and  $S$  does not satisfy  $\psi$

or:

(c) There is a formula  $\pi$  and a formula  $\psi$  such that  $\phi = \lceil \pi \vee \psi \rceil$  and either  $S$  satisfies  $\pi$  or  $S$  satisfies  $\psi$

or finally:

(d) There is a number  $k$  and a formula  $\psi$  such that  $\phi = \lceil (\forall v)\psi \rceil$ , where  $v$  is the  $k$ th variable, and every infinite sequence which differs from  $S$  in at most the  $k$ th place satisfies  $\psi$ .