Application Example 14

(FOSM analysis for functions of many variables)

PROBABILISTIC ANALYSIS OF FOUNDATION SETTLEMENT

Mechanical Model

Consider a footing resting on a compressible soil stratum of depth z_0. Below z_0, the soil may be considered incompressible (e.g. there is bedrock or the stresses induced by the foundation are small enough that soil deformation may be neglected). An important problem is to evaluate the amount of settlement.

The total settlement D may be found by adding the vertical strain of the soil elements along the vertical line through the center of the footing. This gives

$$D = \int_0^{z_o} \frac{\Delta\sigma(z)}{M(z)} dz \qquad (1)$$

where $\Delta\sigma(z)$ is the increment in the vertical stress at depth z due to the foundation load and M(z) is the elastic modulus of the soil at that depth (a deformability parameter of the soil). For simplicity, we divide the compressible soil stratum into n layers of equal thickness (z_0/n) and replace the integral in Eq. 1 with the discrete summation

$$D = \frac{z_o}{n} \sum_{i=1}^{n} \frac{\Delta\sigma_i}{M_i} \qquad (2)$$

where $\Delta\sigma_i = \Delta\sigma(z_i)$, $M_i = M(z_i)$, and $z_i = (i - 0.5)\, z_0/n$ is the average depth of layer i. If the footing is circular of radius R and the load per unit area transmitted by the footing is uniform with value P, then the stress increment profile $\Delta\sigma(z)$ is given by (Poulos and Davis, 1974, p. 43)

$$\Delta\sigma(z) = P\left\{1 - \left[\frac{1}{1+(R/z)^2}\right]^{3/2}\right\} \tag{3}$$

A plot of $\Delta\sigma(z)/P$ as a function of the normalized depth z/P is shown in Figure 1.

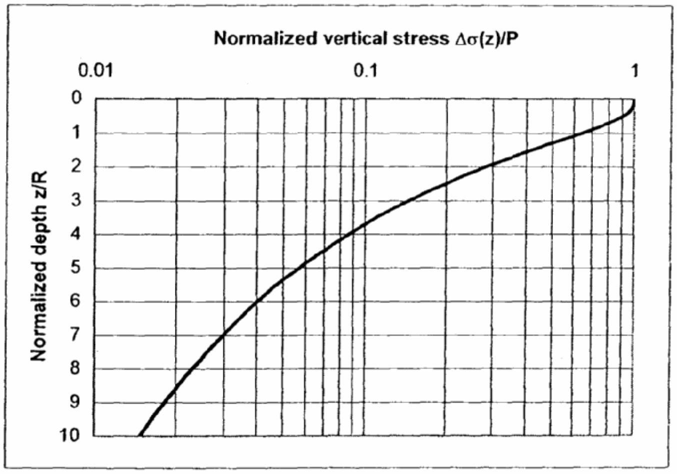

Figure 1. Normalized vertical stress $\Delta\sigma(z)/P$ along the vertical beneath a circular foundation

Analysis of Uncertainty

The main source of uncertainty on the settlement D are the elastic moduli M_i, which in most cases are not accurately known. Suppose that such moduli have mean values m_i, variances σ_i^2, and covariances $\rho_{ij}\sigma_i\sigma_j$, where ρ_{ij} has the form $\rho_{ij} = \rho(|z_i - z_j|)$ and $\rho(\Delta)$ is a "vertical correlation function" of suitable form.

Since D in Eq. 2 is a nonlinear function of the elastic moduli M_i, we calculate in approximation the mean value and variance of D through first-order second-moment (FOSM) analysis. Hence we linearize Eq. 2 with respect to the random variables M_i and then use the exact second-moment propagation formulas for linear functions. Linearization of Eq. 2 around the mean values m_i gives

$$D = \frac{z_o}{n}\left\{\sum_{i=1}^{n}\frac{\Delta\sigma_i}{m_i} - \sum_{i=1}^{n}\Delta\sigma_i\frac{M_i - m_i}{m_i^2}\right\} \qquad (4)$$

Using second-moment analysis with Eq. 4, the following approximations to the mean and variance of D are obtained

$$m_D = \frac{z_o}{n}\sum_{i=1}^{n}\frac{\Delta\sigma_i}{m_i} \qquad (5a)$$

$$\sigma_D^2 = \left(\frac{z_o}{n}\right)^2 \sum_{i=1}^{n}\sum_{j=1}^{n}\Delta\sigma_i\,\Delta\sigma_j\,\rho\left(\left|z_i - z_j\right|\right)\frac{\sigma_i\sigma_j}{\left(m_i m_j\right)^2} \qquad (5b)$$

Problem 14.1

Consider a circular foundation of radius R = 1 meter on a compressible stratum of depth z_o *= 5 meters (notice from Figure 1 that for z/P = 5 the stress increment is already reduced to only 6% of the value just beneath the foundation). Divide the soil stratum into n = 10 layers. Suppose that the stress applied by the foundation is* $P = 10\ Kg/cm^2$ *and that the elastic moduli* M_i *have the following second-moment characteristics:*

mean value $m_i = 10^3\ Kg/cm^2$ *for all i*

coefficient of variation $V_i = \sigma_i/m_i = 0.3$ *for all i*

correlation coefficients $\rho_{ij} = \rho^{|i-j|}$

where ρ *is the correlation coefficient between the elastic moduli of two neighboring layers. Calculate the mean, variance, and coefficient of variation of the settlement D for* ρ *= 0.1, 0.2, ..., 0.9, 1. Plot the coefficient of variation* V_D *against* ρ *and comment on the results.*

The one-dimensional approach described above is approximate, not just due to the linearized analysis, but also because we have ignored the behavior of the soil away from the vertical axis below the foundation center. In reality, the compressibility of the soil at locations away from the vertical through the center affects the state of stress under the foundation and therefore its settlement. For example, if the soil away from the vertical is less compressible than the soil along the vertical, one would expect the foundation to settle less than predicted by the one-dimensional model. The inclusion of three-dimensional effects requires a rather more complicated model and a numerical approach. However, the philosophy is similar to that of the one-dimensional model. For a description of the three-dimensional method and a comparison of results from the two approaches, see Baecher and Ingra (1981).

References

Poulos H. D. and Davis, E. H., *Elastic Solutions for Soil and Rock Mechanics,* Wiley & Sons, 1974.

Baecher, G. B. and Ingra, T. S., "Stochastic FEM in Settlement Prediction," J. of the Geothechnical Engineering Division, ASCE, Vol. 107, No. GT4, pp. 451-463, 1981.