

Problem Set 3: Population Ecology Solutions

1. Two species, A and B, are placed in a limited environment in a laboratory at various initial population densities. 10 different densities were tested. After a month, the population densities of both species were re-measured.

Trial #	Initial densities (t=0)		Densities after one month	
	N _A	N _B	N _A	N _B
1	50	15	55	20
2	100	10	110	20
3 dN_A/dt=0	60	35	60	45
4	20	20	25	15
5 dN_B/dt=0	30	15	40	15
6 dN_A/dt=0	20	25	20	20
7	60	25	65	35
8 dN_B/dt=0	30	40	20	40
9	20	40	15	35
10 dN_A/dt=0	135	5	135	10

(a) Plot the isoclines for Species A and B (use the blank graph on the last page). Explain your method for determining the isoclines.

The isocline for Species A will occur when $dN_A/dt=0$. Hence, points along the isocline will not show a change in N_A over time. This occurs in Trials 3, 6 and 10.

The isocline for Species B will pass through points where N_B doesn't change over time. This occurs in Trials 5 and 8.

Plotting each trial as a vector shows the direction of the arrows in each regions.

See next page for graph.

(b) What kind of interaction do these two species have?

Given the vertical isocline for Species B and the curved isocline for Species A, this is a predator-prey interaction, with A as the prey and B as the predator.

(c) Will it be stable or unstable?

Since the predator isocline passes through the prey isocline to the left of the maximum, this is going to be an expanding set of populations, with no stable point. Since the predator is so efficient at catching prey, it will likely wipe out the populations and the prey population will crash, followed by a crash in the predator population.

- (d) If it is unstable, what factors would make it stable? If it is stable, what factors would make it unstable?

This could become a stable interaction if: (providing one of these answers, or another plausible one, is fine)

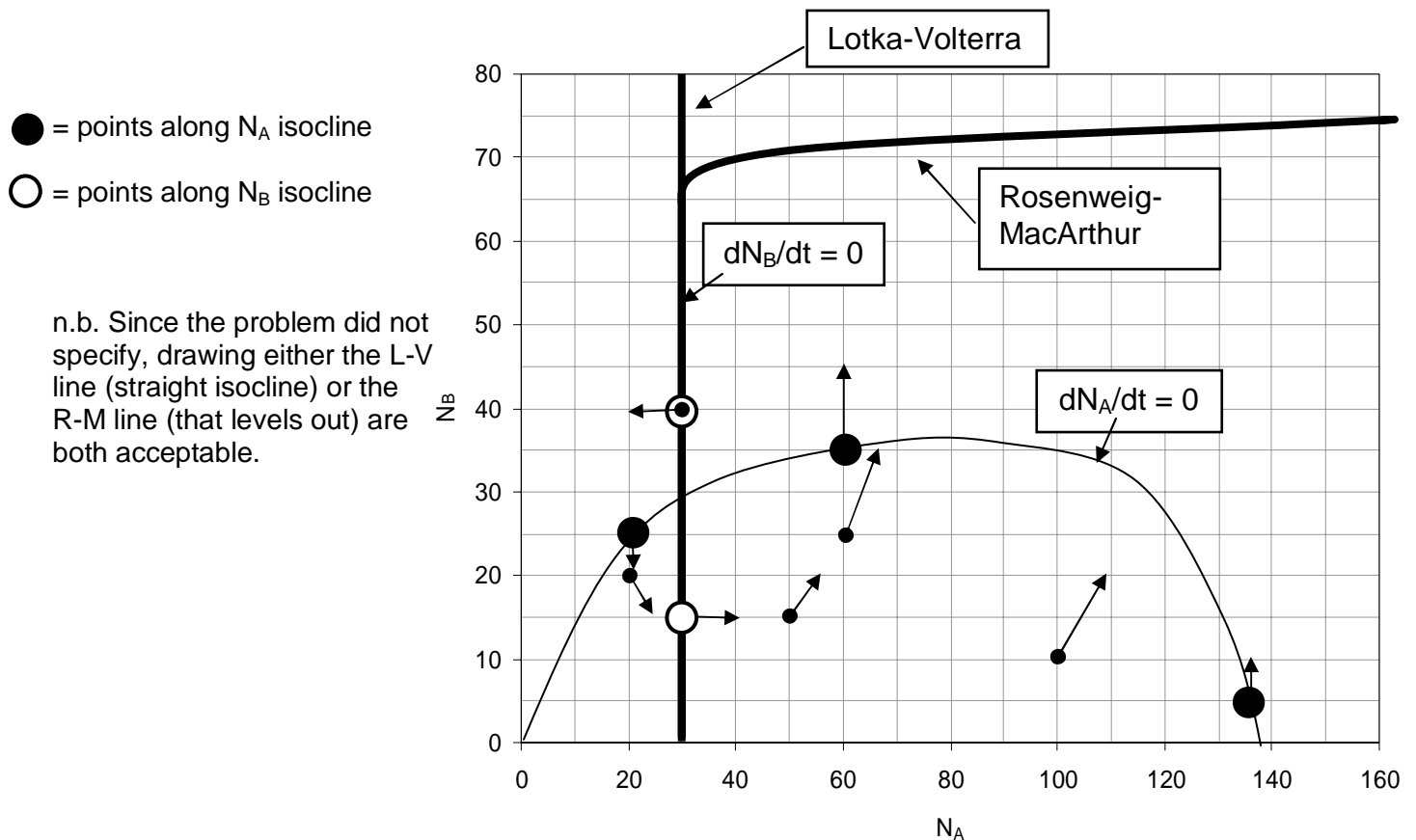
- (1) the predator became less efficient at catching prey
- (2) if the predator became very limited by space or another resource such that the curve hit a maximum below the maximum of the prey isocline
- (3) if the prey carrying capacity became a lot lower

- (e) Approximate the maximum population density possible for Species A, in the absence of Species B.

Based on where the isocline for Species A hits the x-axis, K_A is approximately 138-140.

- (f) Can you determine the maximum density for Species B in the absence of Species A? If so, estimate this maximum density. If not, why not?

No. In this case, since Species B is the predator, we would assume that in the absence of Species A, the maximum density for Species B would be zero. If we assume that Species B could switch to another food source, there is no information provided in the problem about alternative food supplies for Species B, so there is no way to determine the maximum density.



2. Now, you repeat this experiment with two more species, C and D. You start with the same initial population densities as in your previous experiment, and again, observe how the populations change after one month. Using these data, answer the same questions as in Question 1 for Species C and D.

Trial #	Initial densities (t=0)		Densities after one month	
	N_C	N_D	N_C	N_D
1 $dN_D/dt = 0$	50	15	60	15
2 $dN_C/dt = 0$	100	10	100	5
3	60	35	55	30
4	20	20	35	30
5	30	15	35	20
6	20	25	25	30
7 $dN_C/dt = 0$	60	25	60	15
8 $dN_D/dt = 0$	30	40	25	40
9 $dN_C/dt = 0$	20	40	20	50
10	135	5	125	3

- (a) Plot the isoclines for Species C and D (use the blank graphs on the last page). Explain your method for determining the isoclines.

The isocline for Species A will occur when $dN_C/dt=0$. Hence, points along the isocline will not show a change in N_C over time. This occurs in Trials 2, 7, and 9.

The isocline for Species D will pass through points where N_D doesn't change over time. This occurs in Trials 1 and 8.

Plotting each trial as a vector shows the direction of the arrows in each regions.

See next page for graph.

- (b) What kind of interaction do these two species have?

Given the intersecting straight lines, we can assume that Species C and D have a competitive interaction.

- (c) Will it be stable or unstable?

The intersection point will be unstable. The vectors in two of the four regions point away from the intersection point of the two lines, making that an unstable equilibrium point. The only stable interactions occur when $N_C = K_C$ or when $N_D=K_D$, which means these 2 populations cannot coexist.

- (d) If it is unstable, what factors would make it stable? If it is stable, what factors would make it unstable?

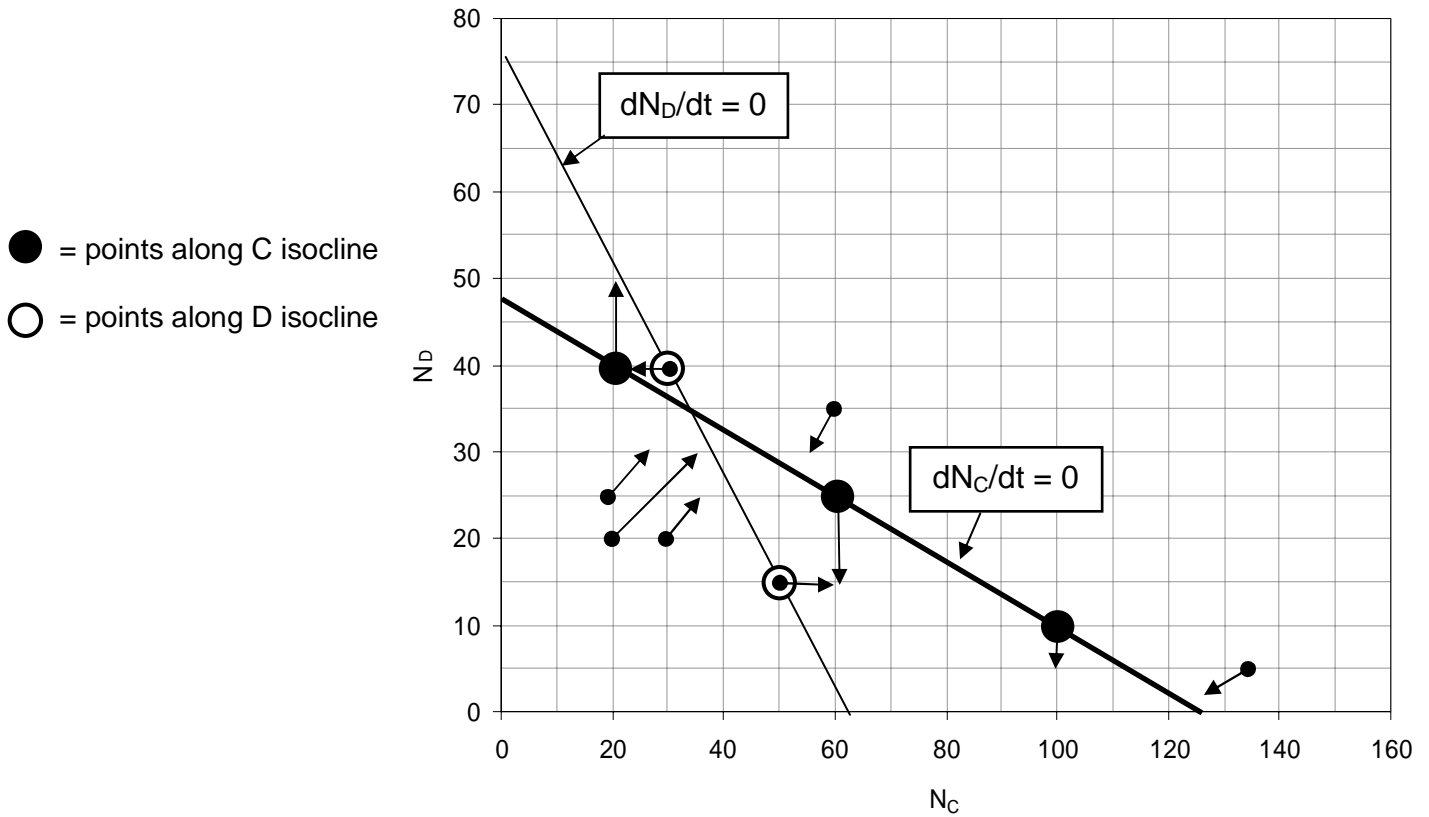
Changing the carrying capacity for one of the species, or changing the α or β value for their interaction. (To make this interaction stable, K_c must be less than K_D/β , and K_D must be less than K_C/α).

- (e) Approximate the maximum population density possible for Species C, in the absence of Species D.

Based on where the isocline for Species C hits the x-axis, K_C is approximately 125.

(f) Can you approximate the maximum density for Species D? If so, what is the value? If not, why not?

Based on where the isocline for Species D hits the y-axis, K_D is approximately 76.



3. Below are mortality and fecundity data for two populations.

x = age (at beginning of age interval)

n_x = number alive at age x

q_x = per capita mortality during the age interval from x to $x+1$ (i.e. fraction that die before reaching next age interval)

l_x = proportion of organisms surviving from the start of the life table to age $x = n_x / n_0$

b_x = per capita birth rate for individuals in age interval.

Species A:

x (yrs)	q_x	l_x	n_x	b_x
0	0.19	1.0	5000	0
1	0.25	0.81	4050	0
2	0.21	0.6075	3038	0
3	0.17	0.4799	2400	0.4
4	0.2	0.3983	1992	0.9
5	0.24	0.3186	1593	0.5

Species B:

x (yrs)	q _x	l _x	n _x	b _x
0	0.99	1.0	2200	0
1	0.2	0.01	22	0
2	0.12	0.008	17.6	0
3	0.08	0.00704	15.488	0
4	0.09	0.006477	14.24896	0
5	0.15	0.005894	12.96655	200

n.b. In class, we mentioned that life tables are often expressed in terms of daughters born per female. In this case, assume that b_x is expressed per individual (not per female).

- (a) Fill in the numbers in the columns l_x and n_x. (We didn't specifically discuss l_x in lecture, but your book describes l_x). Remember that while q_x describes mortality from one interval to the next, l_x describes the fraction surviving compared to the initial number.

Sample calculations:

For x=1 in Species A:

$$l_x = (l_{x-1}) * (1 - q_{x-1}) = 1.0 * (1 - 0.19) = 0.81$$

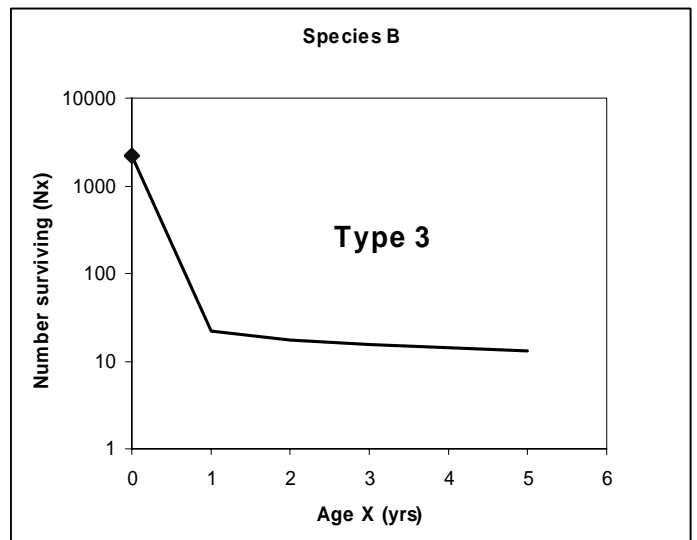
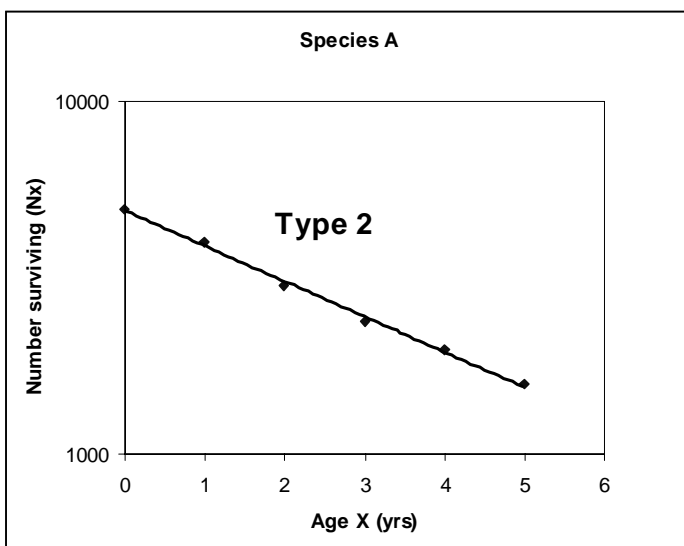
$$n_x = n_0 * l_x = 5000 * 0.81 = 4050$$

For x=2 in Species A:

$$l_x = (l_{x-1}) * (1 - q_{x-1}) = 0.81 * (1 - 0.25) = 0.6075$$

$$n_x = n_0 * l_x = 5000 * 0.6075 = 3037.5$$

- (b) For each species, draw the survivorship curve and state whether it exhibits a Type I, II or III survivorship curve (remember the y axis in a survivorship curve is logarithmic).



(c) Let's say that while you very carefully collected your data, you were less careful with labeling each of the data sets. You know that these data come from either sparrows, salmon or deer. For each Species, state which animal it is likely to be and why.

Species A is likely to be sparrow. Birds typically show Type 2 survivorship curves.

Species B is likely to be salmon. Fish typically show Type 3 survivorship curves.

Another parameter that is frequently considered in population studies is the net productive rate R_0 . R_0 is defined as the number of individuals born per individual per generation. As described in your text, you can calculate R_0 as:

$$R_0 = \sum l_x b_x \quad \text{for all age groups}$$

(d) In general, for what value of R_0 will the total number of individuals in a population remain the same?

$R_0 = 1$ when a population is staying exactly constant (one individual to replace each individual), and birth rates exactly equal death rates.

(e) For the above two populations, calculate R_0 . Are these populations at, above or below the replacement rate required for stable population?

For Species A:

$$R_0 = \sum l_x b_x = 0.4799 * 0.4 + 0.3983 * 0.9 + 0.3186 * 0.5 = 0.71$$

Since $R_0 < 1$, this population will be below replacement rate and will be getting smaller.

For Species B:

$$R_0 = \sum l_x b_x = 0.005894 * 200 = 1.18$$

Since $R_0 > 1$, this population will be above replacement rate and will be expanding.