

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**  
**Department of Civil and Environmental Engineering**

**1.020 Ecology II: Engineering for Sustainability**

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**Problem Set 6 – Life Cycle Analysis, Transport, Economics**

**This is a practice set that should not be turned in and will not be graded. Solutions will be posted before the final exam.**

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**1. Life Cycle Assessment of Steel Rolled Sections**

This problem requires you to carry out a life cycle analysis of 10 kg of steel rolled sections. The data attached at the end of the problem set are sufficient to perform the analysis. You should follow the general procedure outlined in the LCA lectures and illustrated with the concrete column example. The tables and variables provided in the attachment should give you enough information to do the following:

- a) Draw the complete steel section production flow chart. Use boxes and arrows to indicate your intermediate processes, draw the system boundaries and include transportation and electricity. Your final output should be a 10kg rolled steel section.
- b) Identify economic and environmental flows and organize them in 2 separate columns, similarly to what you have been shown in class and mark them as a plus if an output and minus if an input.
- c) Construct your Technology and Environmental Matrices and perform the necessary calculations to obtain your environmental flow vector 'g'. What do the elements of 'g' tell you about the impacts of producing 10 kg of rolled steel section.
- d) How does your carbon dioxide result compare with the concrete example shown in class? Remember that you should compare similar functional units.

**2. Diffusion Time Scales**

A crude method of estimation of the times scales  $\Delta t$  of transport processes is to use an order of magnitude argument. From Fick's Law of diffusion in one dimension

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

Making the approximations  $\frac{\partial c}{\partial t} \approx \frac{\Delta c}{\Delta t}$  and  $\frac{\partial^2 c}{\partial x^2} \approx \frac{\frac{\Delta c}{\Delta x}}{\Delta x} \approx \frac{\Delta c}{\Delta x^2}$ ,

We have

$$\frac{\Delta c}{\Delta t} = D \frac{\Delta c}{\Delta x^2}$$

Rearranging, we get

$$\Delta t = \frac{\Delta x^2}{D}$$

If a coffee cup is filled to 10cm with water, with sugar syrup lying still at the bottom of the cup, what is a rough time for the sugar to diffuse up to the surface, without any stirring? The molecular diffusion of sugar in water is about  $10^{-5} \text{ cm}^2/\text{s}$ .

### 3. Radial Turbulent Diffusion

The planar polar version of the Fickian diffusion equation can be written thus:

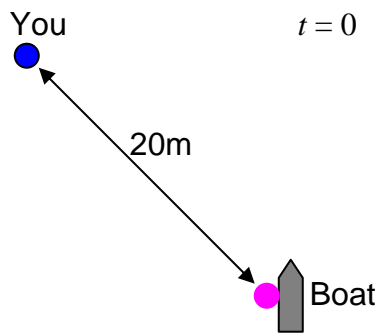
$$\frac{\partial c}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( rD \frac{\partial c}{\partial r} \right)$$

This form is useful for situations where you may treat the diffusion as happening on a flat 2D surface (e.g. shallow water which is vertically well mixed).

a) Show that the following expression satisfies the PDE above.

$$c(r,t) = \frac{m}{4\pi hDt} \exp\left(-\frac{r^2}{4Dt}\right)$$

b) A boat is anchored on a very large tidal flat at high tide (no shoreline nearby for simplicity). You are working about 20m offshore from the boat with a fluorometer that measures dye concentration. The boat suddenly leaks  $m = 1.5\text{kg}$  of dye, which mixes itself quickly over the depth of  $h = 1\text{m}$ , and then starts spreading radially. Using a value of turbulent diffusion  $D = 0.15\text{m}^2/\text{s}$ , calculate the time taken for the dye concentration from the leak to first register 300 micrograms/liter on your fluorometer (at which point your readings will be off calibration). Hint: you will need to use a graphical method, Excel, a graphical calculator or a MATLAB's `fzero` function to solve the above transcendental equation for  $t$ . Don't start your guess at  $t = 0$ , since the computer won't be able to handle this.



#### 4. Discounting

You are concerned with the payoff from an energy conservation investment (e.g. replacement of your leaky windows with new energy conservative windows). The capital cost of the energy improvement (e.g. the windows) is \$5,000 (today). The savings in heating cost is \$200 per year (in the future).

a) If the window life is 30 years and you can expect to earn 5% per year on your investment if you keep it in the bank what is the net cost (savings) of the energy conservation investment?

b) What does the annual savings have to be for the conservation investment to be “break even” (i.e. for the net present value of the savings to be \$5,000)?

Such problems are often solved by comparing the present values of the capital cost and the future savings. The present value of income (or savings)  $I_t$  obtained in a single year  $t$  years in the future is:

$$I_0 = (1+r)^{-t} I_t$$

where  $r$  is the interest rate (expressed as a fraction)

The total present value obtained over  $N$  years is:

$$I_0 = \sum_{t=1}^N (1+r)^{-t} I_t$$

If all the annual incomes are all the same value  $I$  the present value is:

$$I_0 = I \sum_{t=1}^N (1+r)^{-t} = I \left[ \frac{(1+r)^N - 1}{r(1+r)^N} \right]$$

For this problem the investment is justified if the present value  $I_0$  exceeds the capital cost of \$5,000.

## DATA FOR PROBLEM 1 (LIFE CYCLE ANALYSIS)

### STEEL ROLLED SECTION

<b>Inputs</b>	<b>Outputs</b>
Cumulative energy: 1.74 MJ	Carbon dioxide: 0.14 kg
Steel: 1 kg	Dust to air: 0.00005 kg
	Nitrogen oxides: 0.00024 kg
	<b>Rolled section: 1 kg</b>
	Sulphur dioxide: 0.00032 kg

### STEEL

<b>Inputs</b>	<b>Outputs</b>
Coke: 0.00025 MJ	Carbon dioxide 0.0756 kg
Electricity: 0.0219 kWh	Carbon monoxide 0.00473 kg
Ferromanganese: 0.0139 kg	Dust to air: 0.0000475 kg
Ferronickel: 0.04 kg	Ferrochromium: 0.0147 kg
Iron ore: 0.022 kg	Nitrogen oxides: 0.0000125 kg
Iron scrap: 0.126 kg	<b>Steel: 1 kg</b>
Lime: 0.0425 kg	Waste heat: 0.117 MJ
Molybdenite: 0.0175 kg	Waste to landfill: 0.036 kg
Natural gas: 0.0375 MJ	
Pig iron: 0.9 kg	
Transport by 28t truck: 0.0153 tkm	
Transport by rail: 0.108 tkm	
Water: 2.7 kg	

### PIG IRON

<b>Inputs</b>	<b>Outputs</b>
Coal: 0.15 kg	Carbon dioxide: 0.000415 kg
Coke: 0.34 MJ	Carbon monoxide: 0.00134 kg
Electricity: 100 kWh	Dust to air: 0.0000319 kg
Iron ore: 0.15 kg	Nitrogen oxides: 0.0000798 kg
Limestone: 0.01 kg	<b>Pig iron: 1 kg</b>
Pellets: 0.4 kg	Sulfur dioxide: 0.000133 kg
Sinter: 1.05 kg	Waste heat: 0.49 MJ
Transport by 28t truck: 0.00348 tkm	Waste to landfill: 0.0207 kg
Transport by rail: 0.189 tkm	
Water: 6 kg	

### IRON ORE

<b>Inputs</b>	<b>Outputs</b>
Diesel in building machine: 0.0255 MJ	Dust to air: 0.00288 kg
Electricity: 0.00142 kWh	<b>Iron Ore: 0.462 kg</b>

Iron ore (resource): 1 kg	Waste heat: 0.00513 MJ
Water: 0.0115 kg	

## PELLETS

### Inputs

Bentonite:	0.04 kg
Electricity:	0.025 kWh
Iron ore:	1.5 kg
Natural gas:	0.069 MJ
Transport by 40t truck:	0.00401 tkm
Transport by rail:	0.0241 tkm
Water:	0.09 kg

### Outputs

Carbon dioxide:	0.0237 kg
Dust to air:	0.000075 kg
Hydrocarbons:	0.0000223 kg
Nitrogen oxides:	0.000315 kg
Carbon monoxide:	0.00021 kg
<b>Pellets:</b>	<b>1 kg</b>
Sulfur dioxide:	0.000134 kg
Waste heat:	0.531 MJ

## SINTER

Inputs	Outputs
Coke: 1.43 MJ	Carbon dioxide: 0.204 kg
Electricity: 0.01 kWh	Carbon monoxide: 0.0257 kg
Iron ore: 1.5 kg	Dust to air: 0.000206 kg
Lime: 0.05 kg	Hydrocarbons: 0.000137 kg
Natural gas: 0.0363 MJ	Nitrogen oxides: 0.000527 kg
Transport by 28t truck: 0.002 tkm	<b>Sinter: 1 kg</b>
Transport by rail: 0.309 tkm	Sulfur dioxide: 0.00126 kg
Water: 0.5 kg	Waste heat: 1.54 MJ

## TRANSPORTATION 32t TRUCK

Inputs	Outputs
Diesel fuel: 0.29 kg	Carbon dioxide: 0.913 kg
	Carbon monoxide: 0.00126 kg
	Methane: 0.0000216 kg
	Nitrogen oxides: 0.00885 kg
	Particulates: 0.000928 kg
	Sulfur dioxide: 0.000174 kg
	<b>Truck operation: 1 tkm</b>
	Waste Heat: 13.2 MJ

## TRANSPORTATION 40t TRUCK

Inputs	Outputs
Diesel fuel: 0.348 kg	Carbon dioxide: 1.1 kg
	Carbon monoxide: 0.00114 kg
	Methane: 0.0000197 kg
	Nitrogen oxides: 0.00992 kg

	Particulates: 0.000875 kg
	Sulfur dioxide: 0.000209 kg
	<b>Truck operation: 1 tkm</b>
	Waste heat: 15.8 MJ

### TRANSPORTATION BY RAIL

<b>Inputs</b>	<b>Outputs</b>
Diesel fuel: 0.00226 kg	Carbon dioxide: 0.00712 kg
Electricity: 0.0396 kWh	Carbon monoxide: 0.0000357 kg
	<b>Freight train operation: 1 tkm</b>
	Methane: 0.000000294 kg
	Nitrogen oxides: 0.000124 kg
	Particulates: 0.0000748 kg
	Sulfur dioxides: 0.00000136 kg
	Waste heat: 0.245 MJ

### ELECTRICITY

<b>Inputs</b>	<b>Outputs</b>
Hard Coal 0.613 kg	Carbon dioxide: 0.979 kg
Natural Gas 0.25 MJ	Carbon monoxide 0.000125 kg
	Dust to Air: 0.00173 kg
	<b>Electricity 1 kWh</b>
	Hydrocarbons: 0.0000129 kg
	Methane $2.02 \times 10^{-9}$ kg
	Nitrogen oxides: 0.00252 kg
	Sulfur Dioxide 0.00402 kg