

1.040/1.401/ESD.018
Project Management

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Resource Allocation involving Continuous
Variable (Linear Programming)
Part II

Samuel Labi and Fred Moavenzadeh
Massachusetts Institute of Technology

Last Lecture:

- What is a feasible region? How to sketch one.
- How to find corners (vertices) of a feasible region
- Objective functions and Constraints
- Graphical solution of LP optimization problems

This Lecture:

Common Methods for Solving Linear Programming Problems

- Graphical Methods
 - The “Z-substitution” Method
 - The “Z-vector” Method

- Various Software Programs:
 - GAMS
 - CPLEX
 - SOLVER

An Example for Illustrating the Solution Methods

Find the maximum value of the function

$$3X + 10Y$$

subject to the following constraints:

$$X \geq 0$$

$$Y \geq 0$$

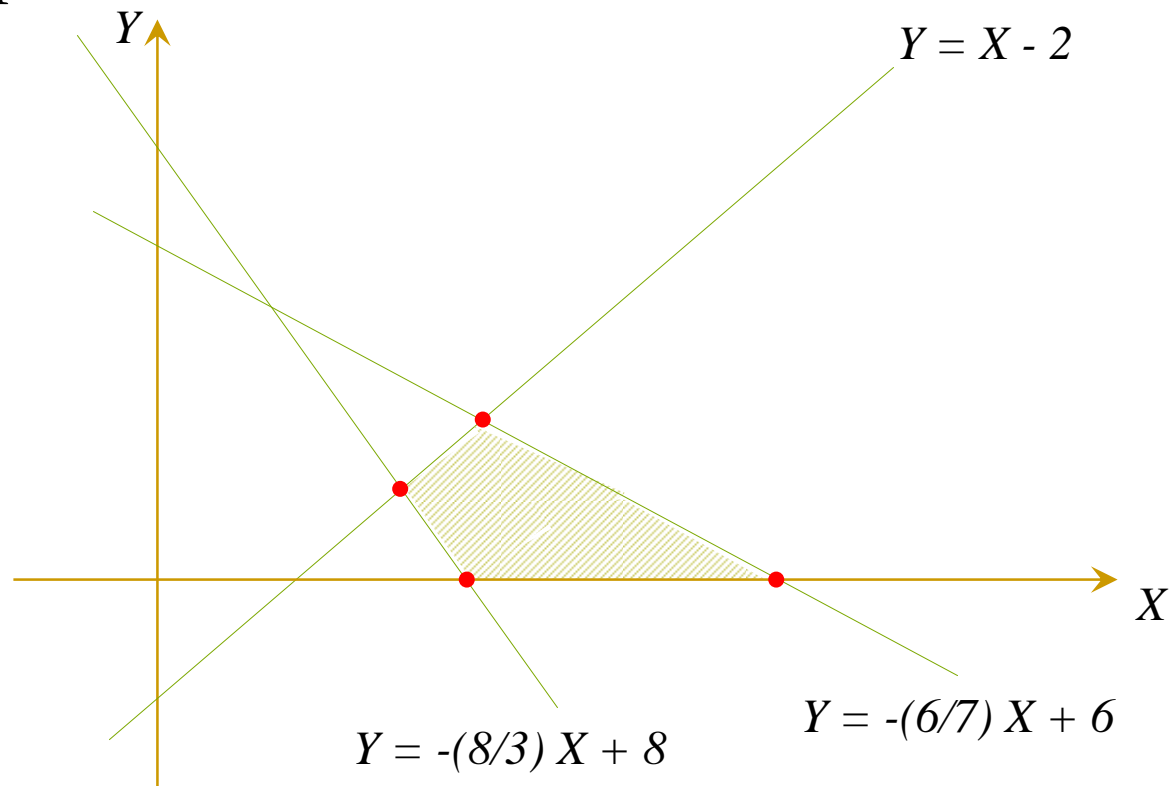
$$Y \geq -(8/3)X + 8$$

$$Y \leq -(6/7)X + 6$$

$$Y \leq X - 2$$

Method 1: Manual or Graphical Solution- The Vertex Technique

There are 2 decision (control) variables: X and Y ,
so problem is 2-dimensional



Method 1: Manual or Graphical Solution- Vertex Technique (cont'd)

Using simultaneous equations (elimination or substitution), or by plotting on a graph sheet, we can determine the vertices and then substitute the various X and Y values into the objective function (W), as follows:

<i>Vertices of Feasible Region (X,Y)</i>	<i>X</i>	<i>Y</i>	<i>W = 3X + 10Y</i>
(2.727, 0.727)	2.727	0.727	15.451
(4.308, 2.307)	4.308	2.308	36.004
(3, 0)	3	0	9.000
(6, 0)	6	0	18.000

← **Optimal Solution**

Method 2:

Manual or Graphical Solution- The “Perpendicular Line” Method

2a. Manual “perpendicular line” method:

1. Find the vertices of the feasible region
2. For each vertex, determine the shortest distance between the origin, and the point where the W vector (the vector representing the objective function (W)) meet the perpendicular line from the vertex.
- 3a. If we seek to maximize W , then the vertex with the greatest linear distance from W is the optimal solution
- 3b. If we seek to minimize W , then the vertex with the shortest distance from W is the optimal solution

Method 2:

Manual or Graphical Solution- The “Perpendicular Line” Method

Note: Shortest distance between any point (a,b) and the line $pX + qY + r = 0$

can be calculated using the formula:

$$\text{Shortest Dist} = \text{sqrt}[(a-V)^2 + (b-U)^2]$$

Where

$$V = [-pr + (q^2)a - pqb] / [p^2 + q^2]$$

$$U = [-qr - pqa + (p^2)b] / [p^2 + q^2]$$

Method 2 (continued):

Manual or Graphical Solution- The “Perpendicular Line” Method

Manual “perpendicular line” method (continued):

For the given example, we seek the distance of each vertex to the vector $\mathbf{W} = 3X + 10Y$

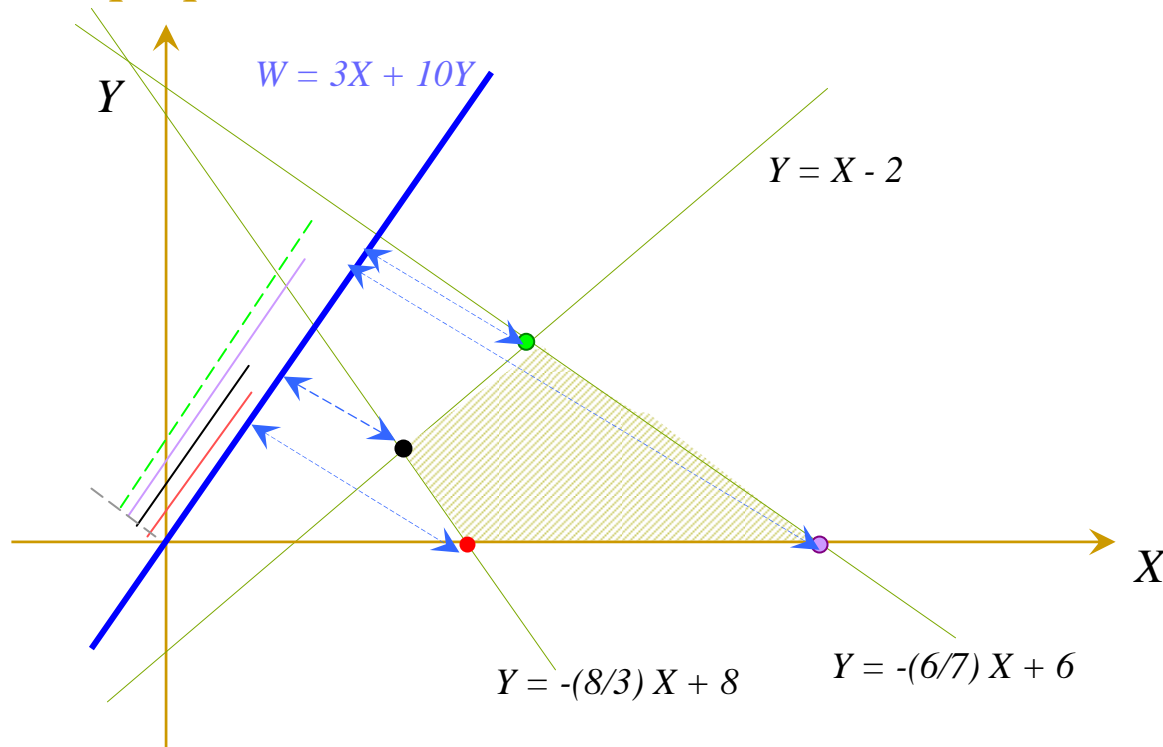
<i>Vertices of Feasible Region (a,b)</i>	<i>a</i>	<i>b</i>	<i>Distance of Meeting Point (of Z-vector and perpendicular line) from Origin</i>
(2.727, 0.727)	2.727	0.727	
(4.308, 2.307)	4.308	2.308	
(3, 0)	3	0	
(6, 0)	6	0	


Optimal Solution

Method 2 (continued):

Manual or Graphical Solution- The “Perpendicular Line” Method

2b. Graphical “perpendicular line” method:



Then simply measure the distances and select the vertex whose intersection with the W vector has the greatest distance from the origin (note here that the green broken line is longest), so the green vertex gives the optimum.

Method 3: Simultaneous Equations

In this method, the vertices of the feasible region are determined as follows:

- employ the technique of substitution or elimination to solve the constraints simultaneously
- Then plug in the vertex values (i.e., value of the decision variables) into the objective function
- Determine which vertex (set of decision variables) yields the optimal value of the objective function.

Method is easy when there are few decision variables and even fewer constraints.

Method 4: Using Linear Algebra (Matrices)

In this method, the vertices of the feasible region are determined as follows:

- Set up the objective function and constraints as a set of linear algebra equations
- Develop the corresponding matrices.
- Solve the set of linear equations using vector algebra. This yields the optimal value of the objective function.
- Simplex method can be employed to help in rapid solution of the matrix

Method is easy when there are few decision variables and even fewer constraints.

Method 5: Using GAMS Software

This program asks you to specify the objective functions and the constraints (these constitute the “input file”)

After running, it gives you the following:

1. A copy of the input file
2. The desired optimal value of the objective function
3. The optimal values of the control variables
4. The model statistics (types and number of equations, variables and elements)
5. Report Summary: -
 - Whether an optimal solution was found
 - whether the solution is feasible
 - whether the solution is bounded

Method 5: Using GAMS Software

Find the maximum value of the function

$$3X + 10Y$$

subject to the following constraints:

$$X \geq 0$$

$$Y \geq 0$$

$$Y \geq -(8/3) X + 8$$

$$Y \leq -(6/7) X + 6$$

$$Y \leq X - 2$$

The Input GAMS file for the given problem is provided on next page.

Method 5: Using GAMS Software – SAMPLE OUTPUT

positive variables

```
x "x- variable",  
y "y- variable";
```

```
free variable  
Z "Z-variable";
```

equations

```
OBJ "Objective function",  
C1  "constraint 1",  
C2  "constraint 2",  
C3  "constraint 3",  
C4  "constraint 4",  
C5  "constraint 5";
```

```
OBJ .. Z =e= 3*x+10*y;  
C1  .. x =g= 0;  
C2  .. y =g= 0;  
C3  .. y =g= -8/3*x+8;  
C4  .. y =l= -6/7*x+6;  
C5  .. y =l= x-2;
```

```
model EXAMPLE2 /all/ ;
```

```
solve EXAMPLE2 using lp maximizing Z ;
```

Please be careful about every single character!

Leaving any small thing out may cause you to have errors in running the program.

Method 5: Using GAMS Software – Explanation of SAMPLE OUTPUT

positive variables

```
x "x- variable",
y "y- variable";
```

```
free variable
Z "Z-variable";
```

equations

```
OBJ "Objective function",
C1 "constraint 1",
C2 "constraint 2",
C3 "constraint 3",
C4 "constraint 4",
C5 "constraint 5";
```

```
OBJ .. Z =e= 3*x+10*y;
```

```
C1 .. x =g= 0;
```

```
C2 .. y =g= 0;
```

```
C3 .. y =g= -8/3*x+8;
```

```
C4 .. y =l= -6/7*x+6;
```

```
C5 .. y =l= x-2;
```

```
model EXAMPLE2 /all/ ;
```

```
solve EXAMPLE2 using lp maximizing Z ;
```

Objective function

$$Z = 3X + 10Y$$

constraints:

$$X \geq 0$$

$$Y \geq 0$$

$$Y \geq -(8/3)X + 8$$

$$Y \leq -(6/7)X + 6$$

$$Y \leq X - 2$$

Method 5: Using GAMS Software

In Class Demo
of GAMS

Method 6: Using MS Solver

Steps:

1. Construct a “Constraints” matrix
2. Put in initial (or “seed”) values for your control (or decision) variables
3. Call up “Solver” to determine which values of the control variables give the optimum value of Z and what that optimum is.

Method 6: Using MS Solver (continued)

In Class Demo of
Solver

Method 7: Using C-PLEX

CPLEX:

- One of the most powerful optimization programs in the world
- Runs on many different platforms
- Available on MIT computers?
- Used mainly for linear programming
- For non-linear programming problems, one has to contact vendor (ILOG) directly.

Method 7: Using C-PLEX (continued)

CPLEX:

- To run CPLEX,

First re-write all constraints in the following form:

$$ax+by+c = 0$$

$$ax+by+c \geq 0$$

$$ax+by+c \leq 0$$

Method 7: Using C-PLEX (continued)

In Class Demo of
CPLEX

Applications of LP Optimization in Project Management(1)

Project managers are constantly engaged in allocating resources in the most effective manner, within given constraints

Resources: Manpower
 Money
 Materials
 Machinery, etc.

Constraints: Financial
 Physical
 Institutional
 Political, etc.

Applications of LP Optimization in Project Management (2)

Resources:

How many items of type X should be used?

How much money to invest in this project?

How many man-hours should be allocated to that task?

Etc.

Applications of LP Optimization in Project Management (3)

Example of constraints:

Financial: Our budget this year is only \$10,000,000!

Physical: But we have only 35 supervisors!

Our supplier can provide only 125 m³ of concrete/hr

Institutional: Do not operate that noisy machine between school hours (project site is near an elementary school!)

Applications of LP Optimization in Project Management (4)

Note:

In project management, the variables we may change in order to obtain a certain objective are also referred to as

Design variables, or

Decision variables, or

Control variables

Constraints are expressed in terms of the decision variables.

Remember that the number of decision variables dictates the number of dimensions of the optimization problem.

Applications of LP Optimization in Project Management (5)

Typical Objectives (Goals):

General: Maximize profit

Minimize Project duration

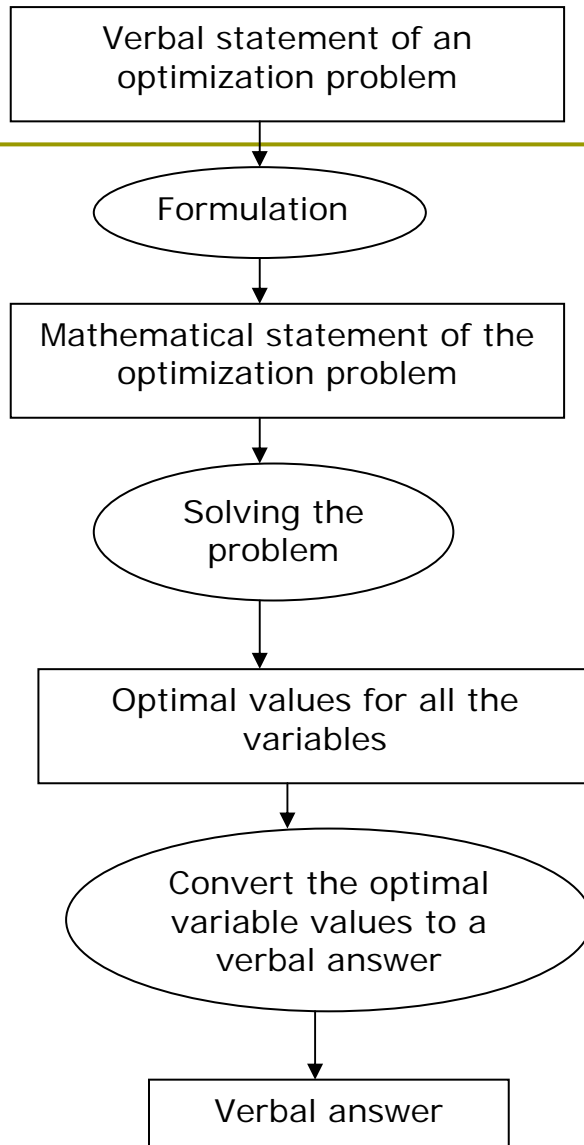
Maximize Worker productivity

Maximize Economic Efficiency (B/C ratio, NPV, etc.)

Minimize Maintenance and Operational Costs

Maximize Safety, Mobility, Aesthetic Appeal

Minimize all Costs incurred over Cash Flow Period



General
Steps in
Typical
Problem

Some standard definitions

Control variables / Decision variables (x_1, x_2, \dots, x_n)

A control variable is a term used to designate any parameter that may vary in the design, planning, or management process.

Objective function

$f(x_1, x_2, \dots, x_n) = f(x)$ is a single-valued function of the set of decision variables (x)

Constraint conditions

These are the mathematical notation of the limitations places upon the problem (e.g., financial, physical, institutional)

Some standard definitions (cont'd)

Feasible solution space

Any combination of decision variables that satisfies the set of constraint conditions is called feasible solution space

Optimum solution

It is a feasible solution that satisfies the goal of the objective function as well.

The optimal solution is a statement of how resources or inputs should be used to achieve the goal in the most efficient and effective manner

Some more interesting stuff about LP Optimization Problems

- A constraint is said to be redundant if dropping the constraint does not change the feasible region. In other words, it does not form a unique boundary of the feasible solution space
- A binding constraint is a constraint that forms the optimal corner point of the feasible solution space

Some more interesting stuff about LP Optimization Problems (2)

Sometimes, the decision variables are all integers, then the problem becomes an **integer programming** problem

Sometimes, we cannot have exact solutions to a programming problem, then we use techniques that are collectively called **heuristic programming**.

Some more interesting stuff about LP Optimization Problems (3)

Integer Programming- when the decision variables are positive integer numbers

Binary Programming – when the decision variables take on only values of either 1 or 0 (examples, “yes” or “no”, “married” or “not married”).

Also called Zero-one Programming.

Mixed Programming – when some decision variables are continuous while others are discrete.

Some more interesting stuff about LP Optimization Problems (4)

Non-linear Programming- when the constraints are non-linear. The solution approach to non-linear optimization problems is different from that of LP problems.

For example, the optimal solution is not necessarily a vertex of the feasible region.

Some more interesting stuff about LP Optimization Problems (6)

Note:

Linear Programming- is only one of many tools that can be used for solving **continuous-variable** resource allocation problems in project management.

Other tools include

- Calculus
- Trial and Error
- Simulation (typically with aid of computer)