Probabilistic Planning Part I

1.040/1.401 – Project Management

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Recall …

Project Management Phases

Project Evaluation

Project Finance

Project Organization

Project Cost Estimation

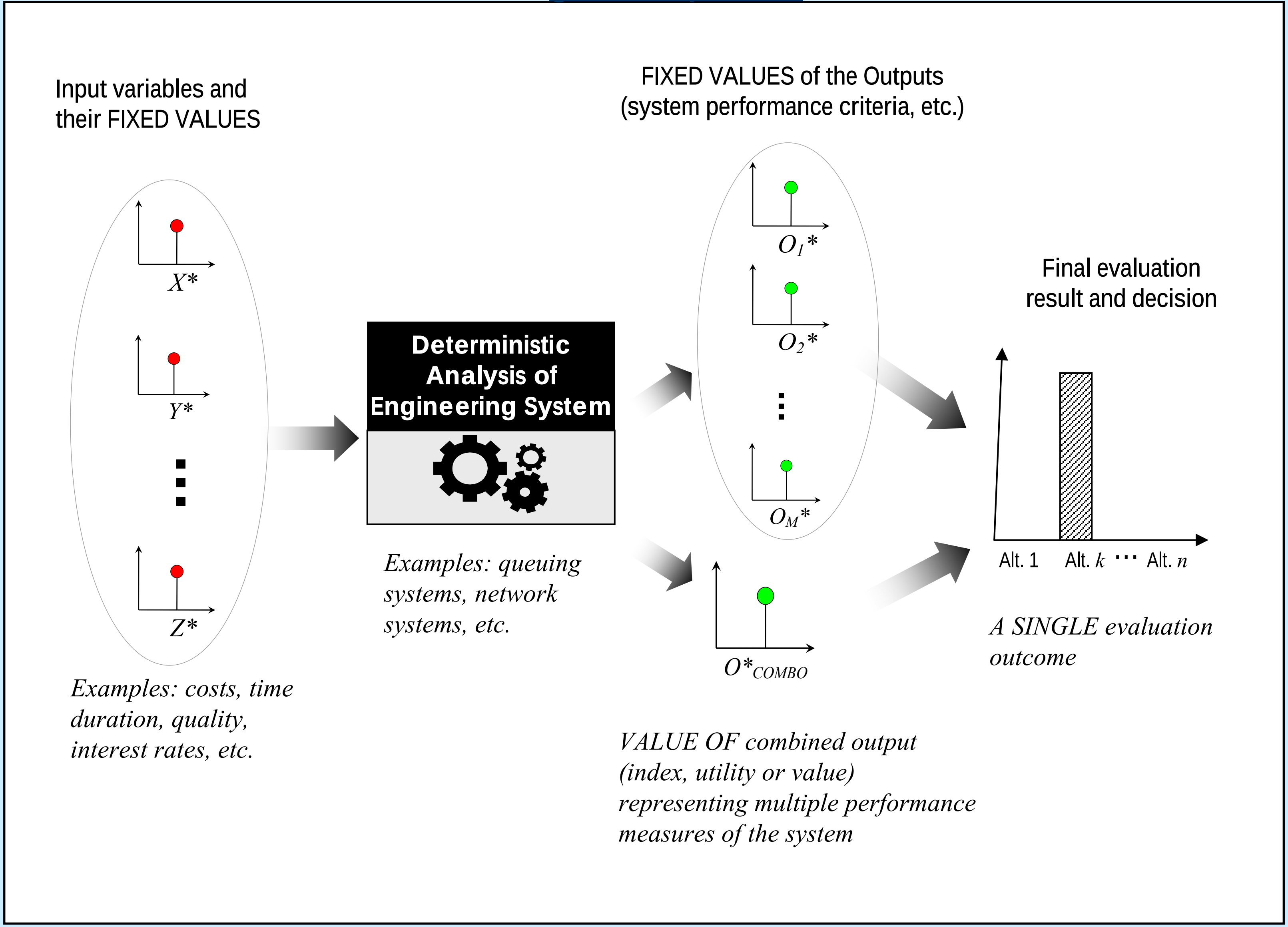
Project Planning and Scheduling

Outline for this Lecture

- Deterministic systems decision-making – the general picture
- Probabilistic systems decision-making – the general picture
  – the special case of project planning
- Why planning is never deterministic
- Simplified examples of deterministic, probabilistic planning
  - everyday life
  - project planning
- Illustration of probabilistic project planning
- PERT – Basics, terminology, advantages, disadvantages, example

Deterministic Systems Planning and Decision-making

Influence of deterministic inputs on the outputs of engineering systems -- the general picture



But engineering systems are never deterministic!

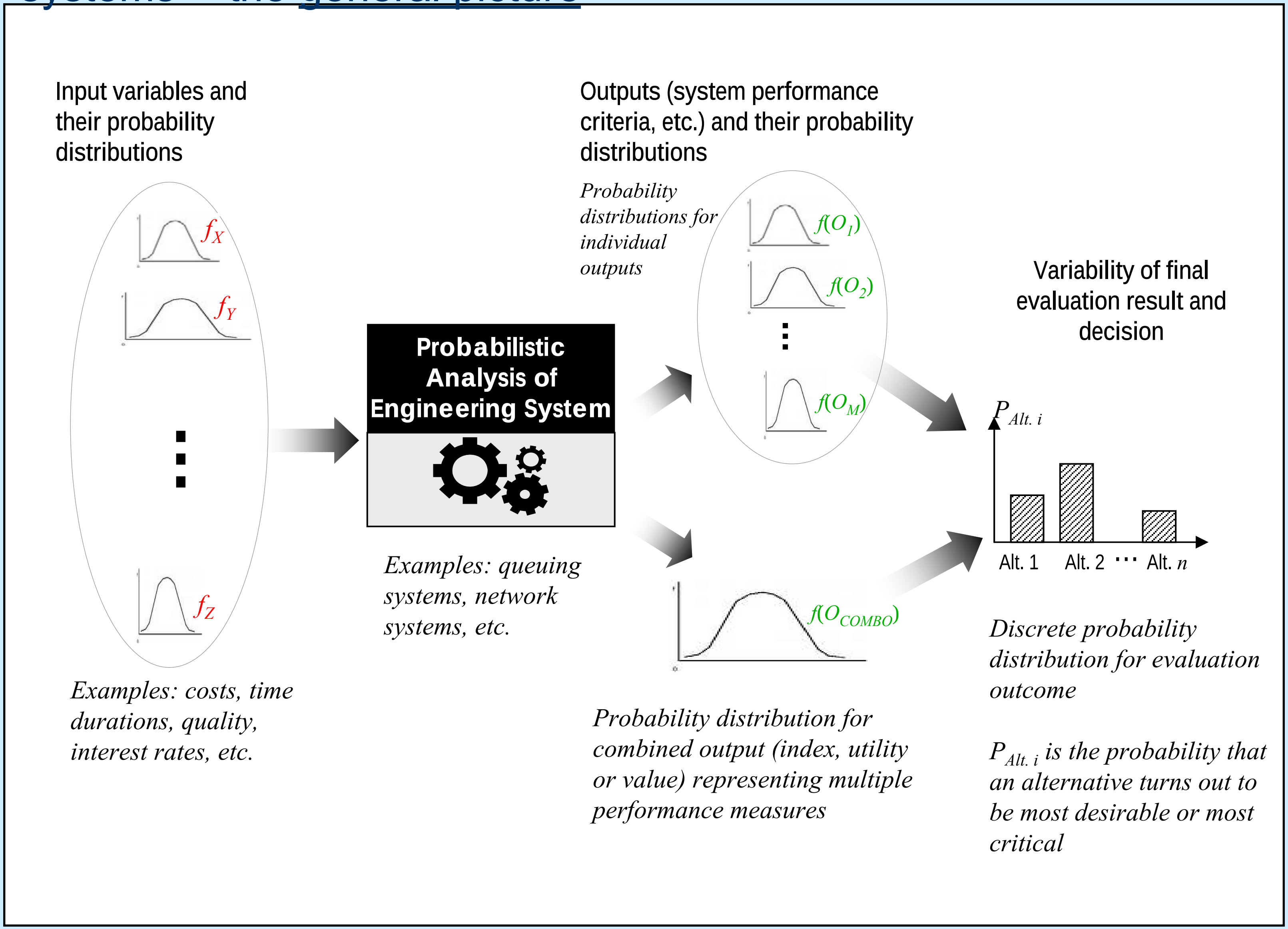
Why?

For sample, for Project Planning Systems …

- Variations in planning input parameters
- Beyond control of project manager
- Categories of the variation factors
  - Natural (weather -- good and bad, natural disasters, etc.)
  - Man-made (equipment breakdowns, strikes, new technology, worker morale, poor design, site problems, interest rates, etc.)
- Combined effect of input factor variation is a variation in the outputs (costs, time, quality)

Why planning is never deterministic -- II

Influence of stochastic inputs on the outputs of engineering systems -- the general picture

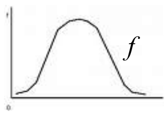


Why planning is never deterministic -- III

Probability distribution?

What exactly do you mean by that?

Input variable and its probability distribution

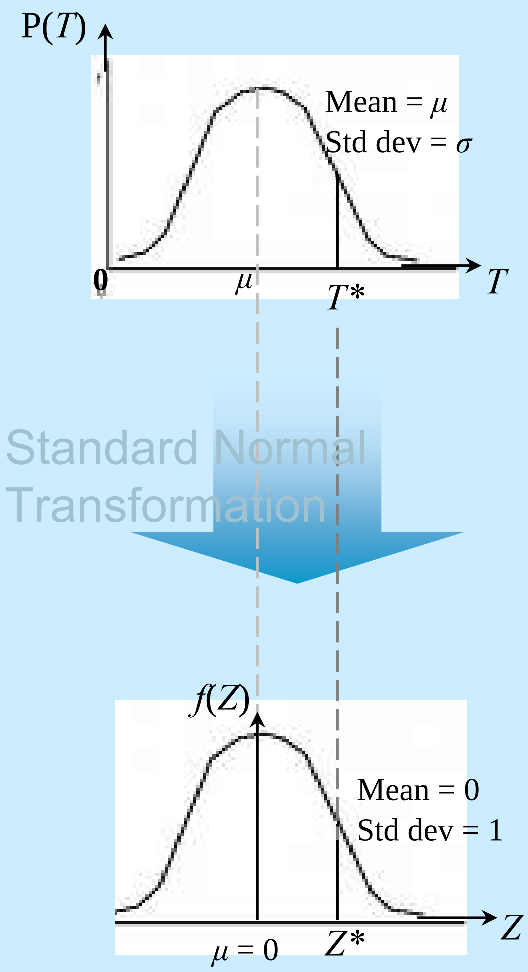


See survey for dinner durations (times) of class members

Probability Distribution for your Dinner Times:

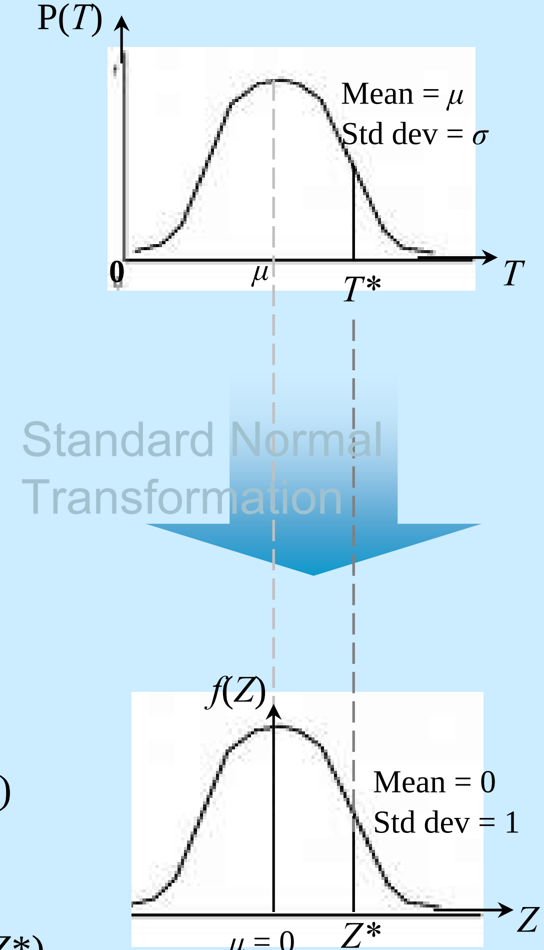
Probability that a randomly selected student spends less than T^* minutes for dinner is:

… less than $T^* = \mathrm{P}(T < T^*) = P\left(\frac{T-\mu}{\sigma} < \frac{T^*-\mu}{\sigma}\right) = P(Z < Z^*)$



Probability that a randomly selected student spends less than T^* minutes for dinner is:

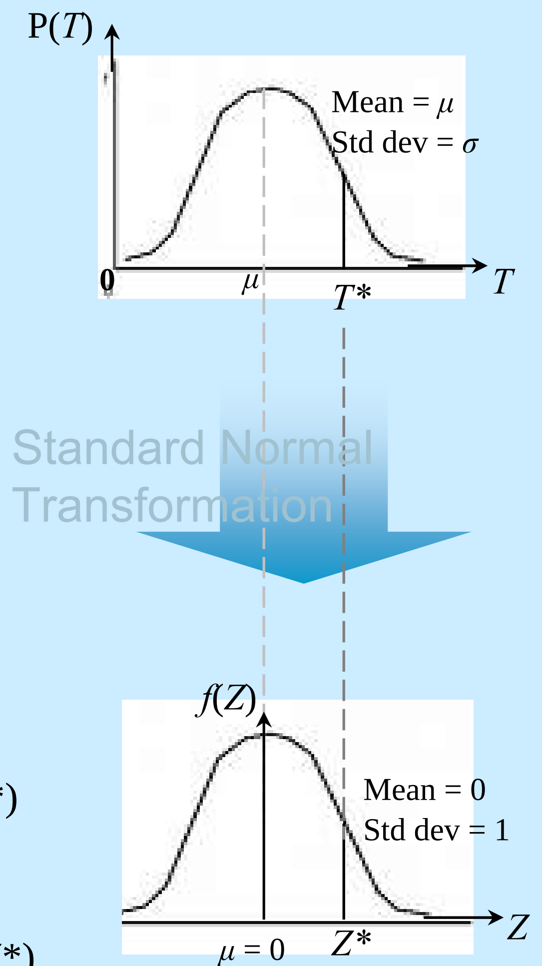
… less than $T_1 = P(T < T^*) = P\left(\frac{T-\mu}{\sigma} < \frac{T^*-\mu}{\sigma}\right) = P(Z < Z^*)$



Probability that a randomly selected student spends …

… less than $T_1 = \mathrm{P}(T < T^*) = P\left(\frac{T-\mu}{\sigma} < \frac{T^*-\mu}{\sigma}\right) = P(Z < Z^*)$

… more than $T_1 = \mathrm{P}(T > T^*) = P\left(\frac{T-\mu}{\sigma} > \frac{T^*-\mu}{\sigma}\right) = P(Z > Z^*)$



Probability that a randomly selected student spends …

… less than T^* = $P(T < T^*)$ = $P\left(\frac{T-\mu}{\sigma} < \frac{T^*-\mu}{\sigma}\right) = P(Z < Z^*)$

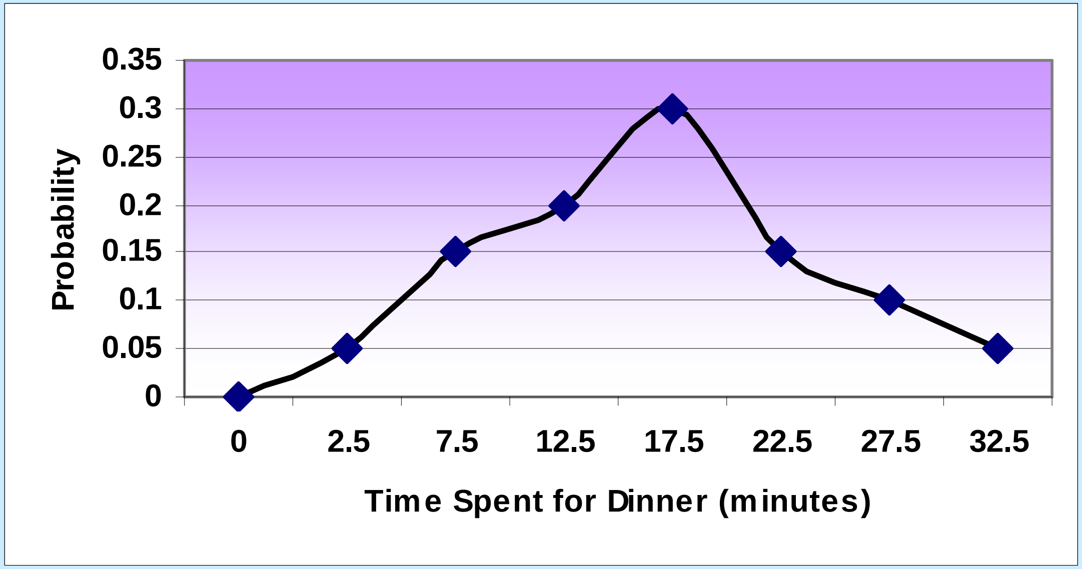
… more than T^* = $P(T > T^*)$ = $P\left(\frac{T-\mu}{\sigma} > \frac{T^*-\mu}{\sigma}\right) = P(Z > Z^*)$

… between T^*_1 and T^*_2 = $P(T^*_1 < T < T^*_2)$ = $P\left(\frac{T_1^*-\mu}{\sigma} < \frac{T-\mu}{\sigma} < \frac{T_2^*-\mu}{\sigma}\right) = P(Z_1^* < Z < Z_2^*)$

Probability is the area under the probability distribution/density curves)

Probability can be found using any one of three ways:

- coordinate geometry
- calculus
- statistical tables



Like-wise, we can build probability distributions for project planning parameters by …

- using historical data from past projects, OR

- computer simulation

And thus we can find the probability that project durations falls within a certain specified range

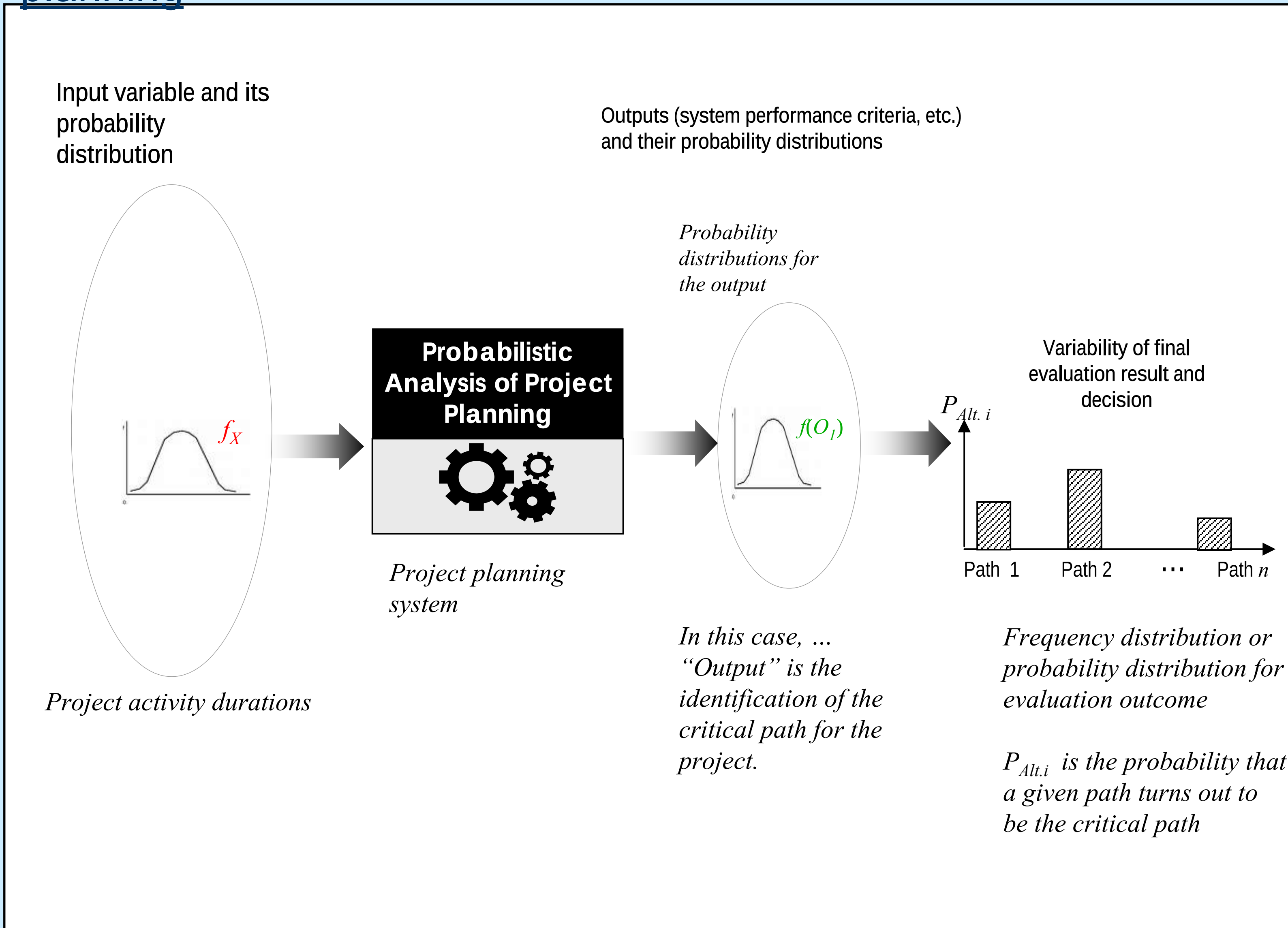
Probabilistic planning of project management systems can involve uncertainties in:

- Need for an Activity (need vs. no need)
- Durations
    - » Activity durations
    - » Activity start-times and end-times
- Cost of activities
- Quality of Workmanship and materials
- Etc.

Probabilistic planning of project management systems can involve uncertainties in:

- Need for an Activity (need vs. no need)
- Durations
  - » Activity durations
  - » Activity start-times and end-times
- Cost of activities
- Quality of Workmanship and materials
- Etc.

Influence of stochastic inputs -- the specific picture of project planning



Probabilistic planning: *An example in everyday life*

Perfectly deterministic

KEY

Activity		
Start Time	Activity Duration	**Finish Time**

SAM Waking up and meditating		
START = 7AM	Duration 1hr	**FINISH =8AM**

SAM Bathing, Breakfast,, Reading,		
START =8AM	Duration 5hrs	**FINISH = 1 PM**

US Meeting in Class For this Lecture		
START= 1PM	Duration1.5hr	**FINISH= 2:30**

YOU Preparing for Classes, etc.		
START= 7 AM	Duration 1 hr	**FINISH= 8AM**

YOU showing up at Other Classes		
START= 8AM	Duration 5 hr	**FINISH=1 PM**

YOU missing the		
START= 1 PM	Duration1.5hr	**FINISH= 2:30**

Probabilistic planning: *An example in everyday life*

Partly deterministic, Partly probabilistic

KEY

Activity		
Earliest Start	Duration of Activity	**Earliest Finish**
Latest Start		**Latest Finish**

SAM Waking up and meditating

ES = 6AM	Duration = 1 hour	**EF =**
LS = 7AM		**LF = 8AM**

→

SAM Bathing, Breakfast,, Reading,

ES =7AM	Duration = 5 hrs	**EF =**
LS = 8AM		**LF = 1 PM**

→

US Meeting in Class

For this Lecture

ES = 12:55	Duration = 1.5 hrs	**EF =**
LS = 1 PM		**LF = 2:30**

YOU Preparing for Classes, etc.

ES = 6:45	Duration = 1 hr	**EF =**
LS = 7AM		**LF = 8AM**

→

YOU showing up at Other Classes

ES = 7:45	Duration = 5 hour	**EF =**
LS = 8AM		**LF = 1 PM**

→

YOU missing the

ES = 6AM	Duration = 1 hour	**EF =**
LS = 7AM		**LF = 8AM**

Probabilistic planning: *An example in everyday life*

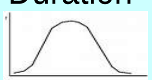
Fully probabilistic

KEY

Activity		
Earliest Start	Duration of Activity	Earliest Finish
Latest Start		Latest Finish

SAM Waking up and meditating

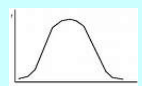
ES = 6AM
LS = 7AM
Duration



$\mu = 1\text{hr}$
$\sigma = 0.25\text{hr}$
EF =
LF = 8AM

SAM Bathing, Breakfast,, Reading,

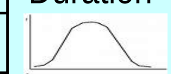
ES =7AM
LS = 8AM
Duration



$\mu = 5$ hr
$\sigma = 0.6\text{hr}$
EF =
LF = 1 PM

US Meeting in Class For this Lecture

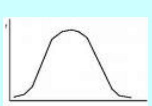
ES = 12:55
LS = 1 PM
Duration



$\mu = 1.5$ hr
$\sigma = 0.31\text{hr}$
EF =
LF = 2:30

YOU Preparing for Classes, etc.

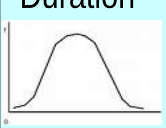
ES = 6:45
LS = 7AM
Duration



$\mu = 1$ hr
$\sigma = 0.15$ hr
EF =
LF = 8AM

YOU showing up at Other Classes

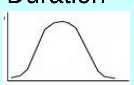
ES = 7:45
LS = 8AM
Duration



$\mu = 5$ hr
$\sigma = 0.35$ hr
EF =
LF = 1 PM

YOU missing the

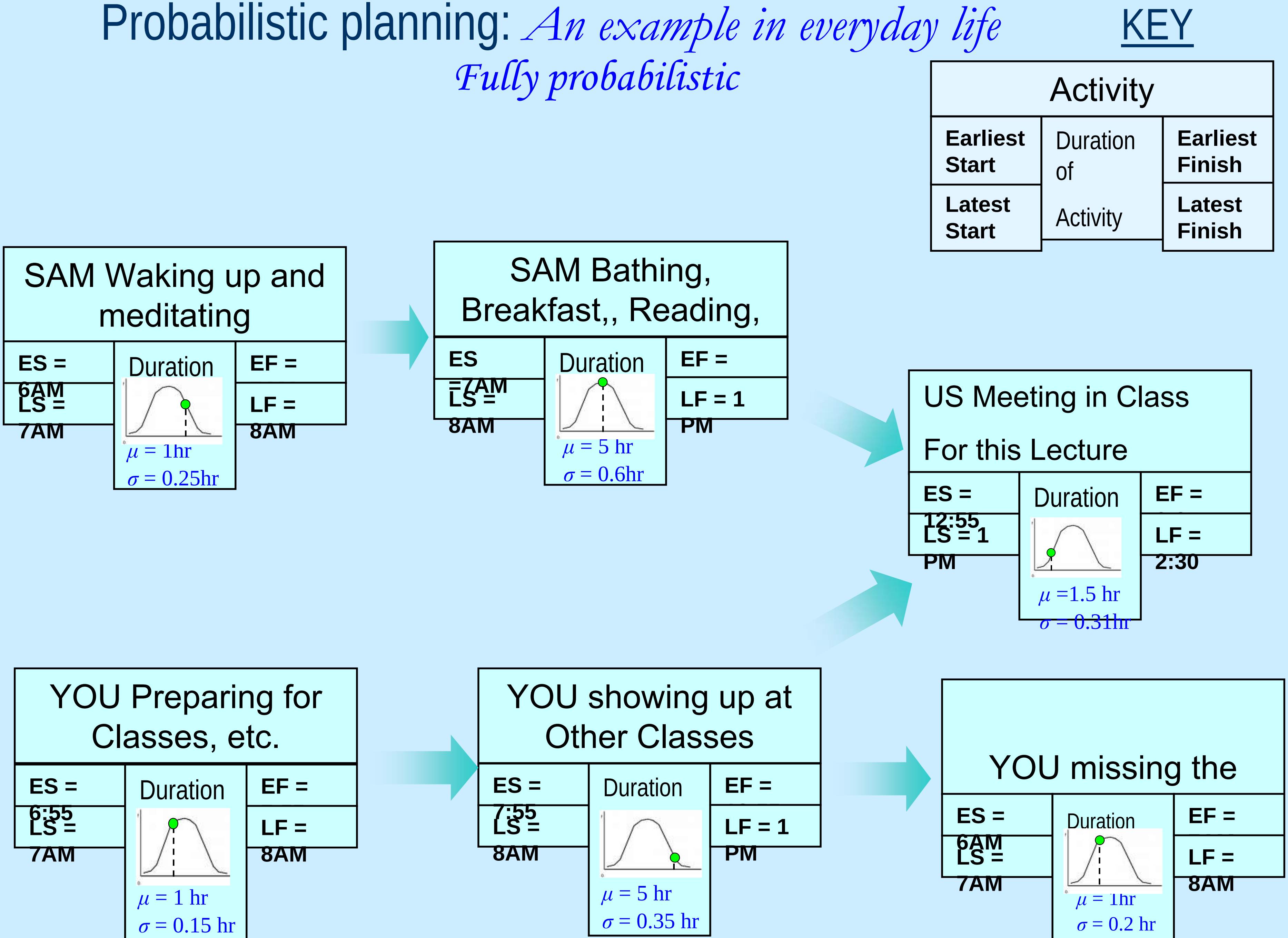
ES = 6AM
LS = 7AM
Duration



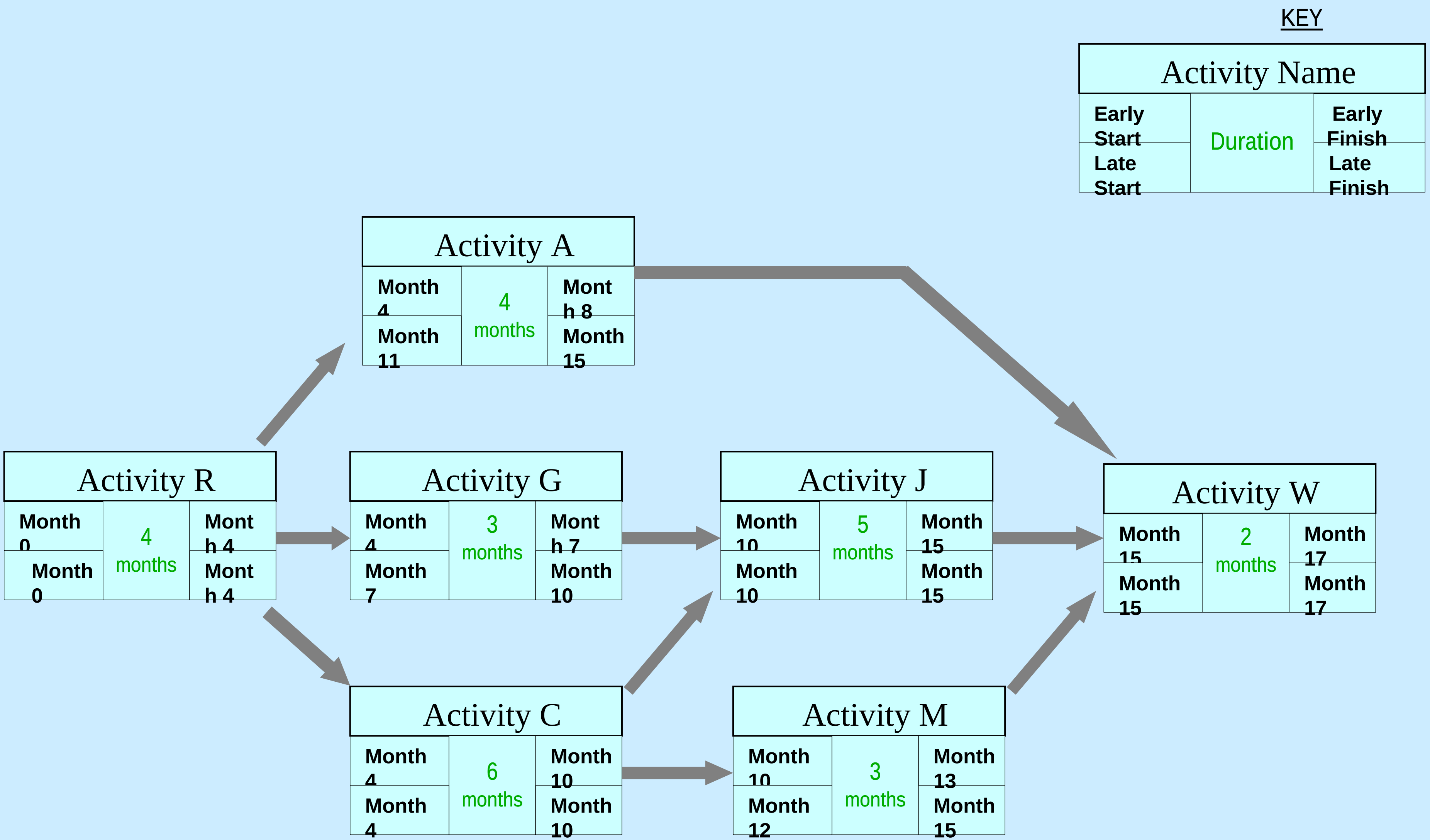
$\mu = 1\text{hr}$
$\sigma = 0.2$ hr
EF =
LF = 8AM

Probabilistic planning: *An example in everyday life*

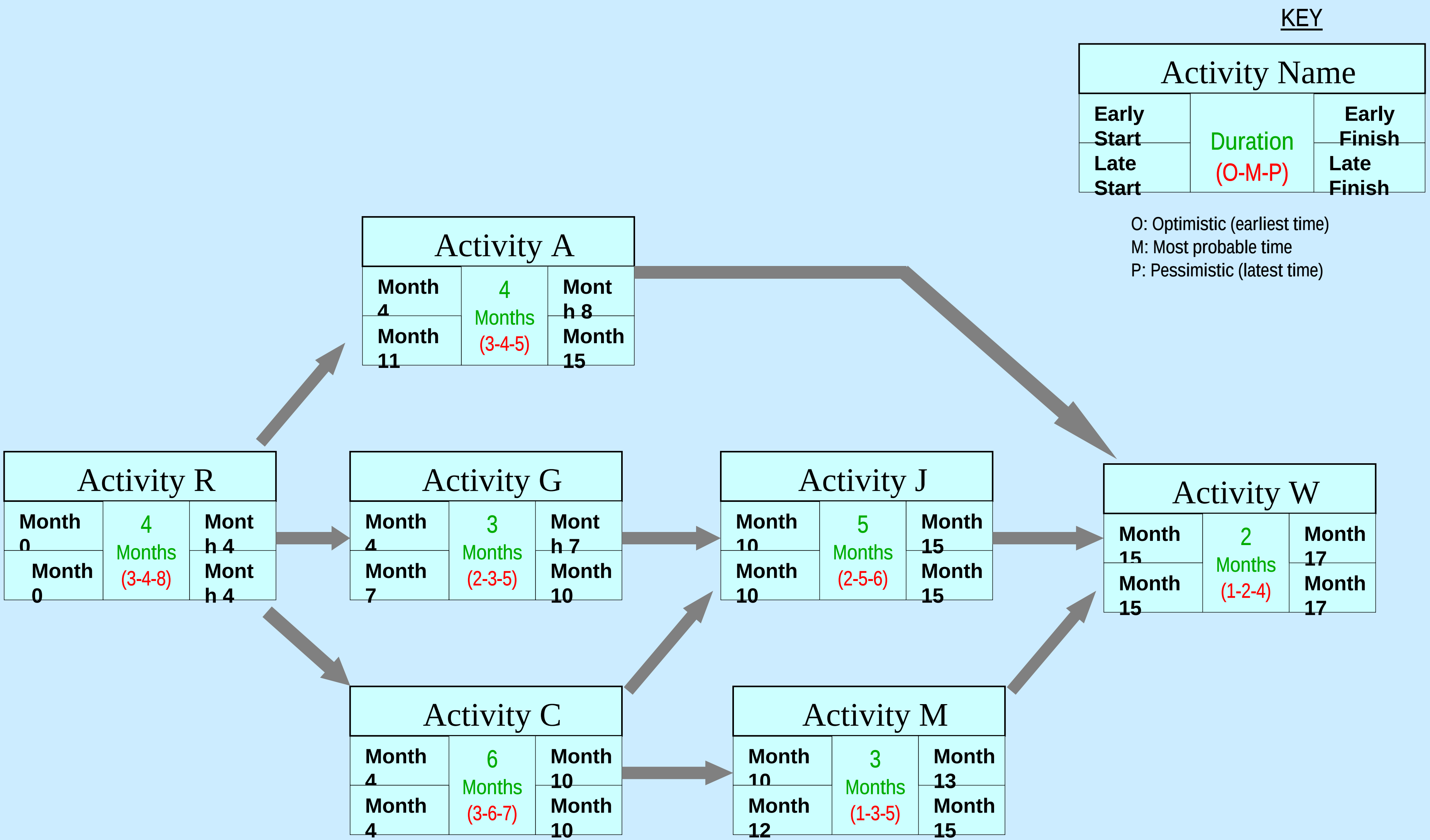
Fully probabilistic



Probabilistic planning: *An example in Project Management*

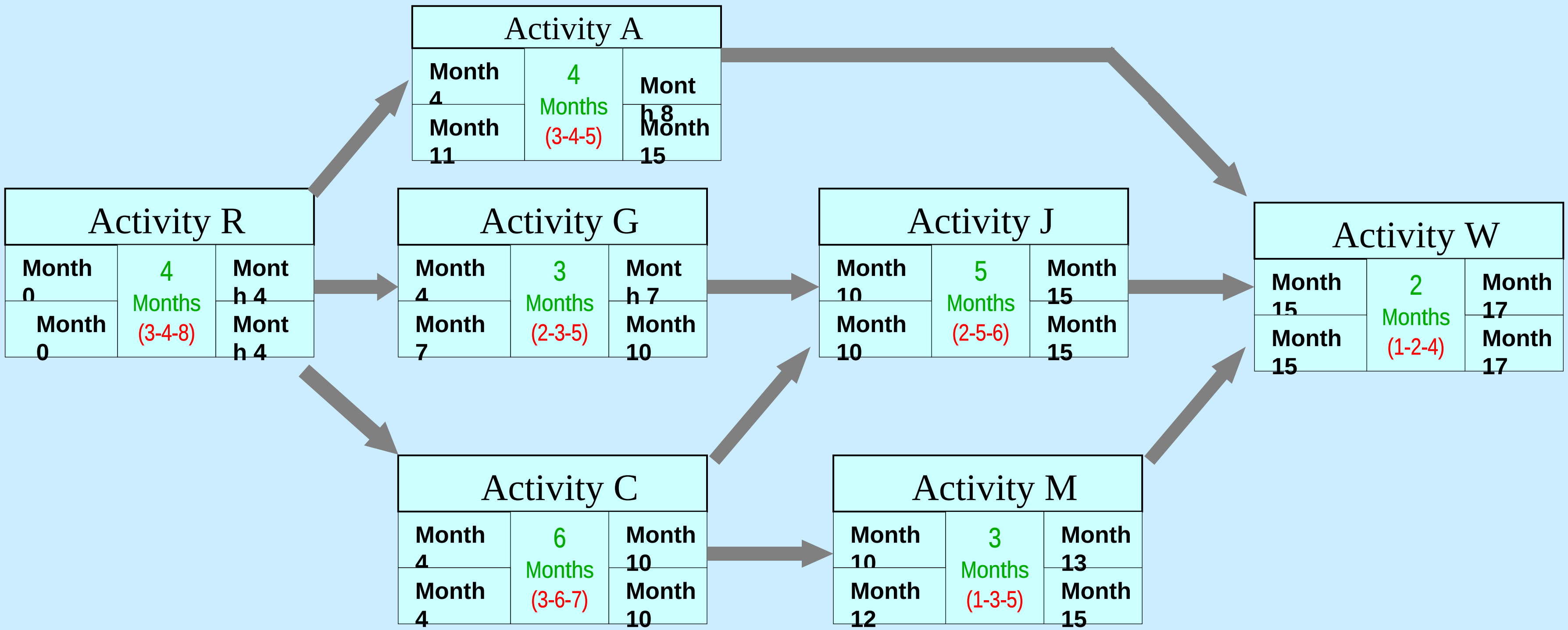


Probabilistic planning: *An example in Project Management*



Probabilistic planning involving activity durations

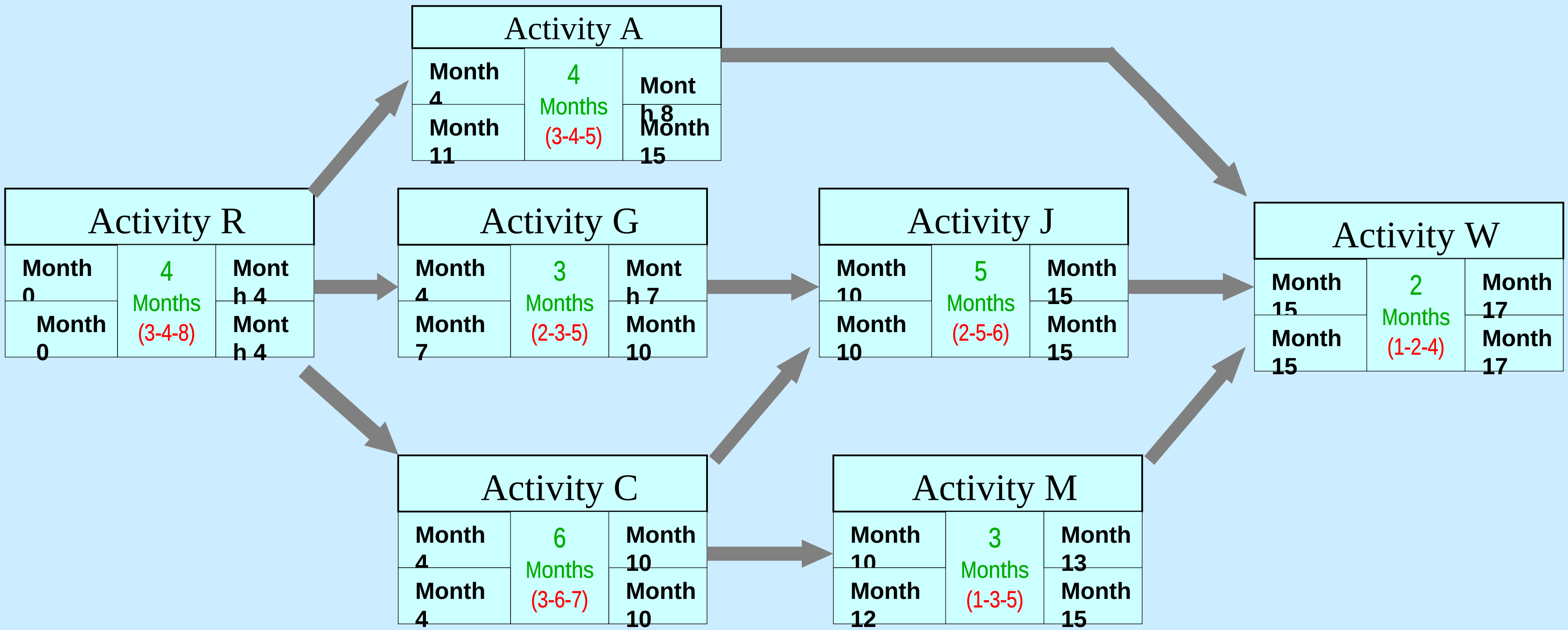
An Illustration



Activity	Optimistic	Most Likely	Pessimistic
R	3	4	8
A	3	4	5
G	2	3	5
C	3	6	7
J	2	5	6
M	1	3	5
W	1	2	4

Probabilistic planning involving activity durations

An Illustration



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J	2	5	6
M	1	3	5
W	1	2	4

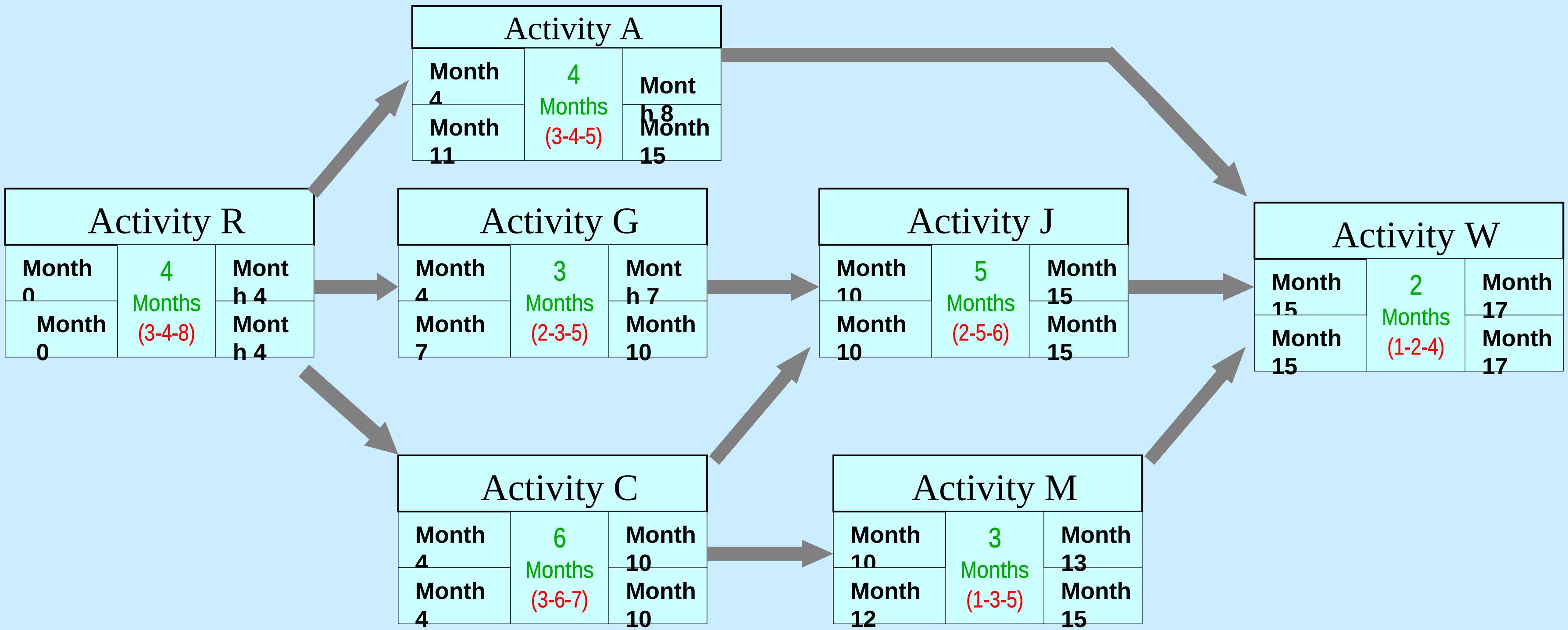
Let's say we have lots of data on the durations of each activity. Such data is typically from:

- Historical records (previous projects)

- Computer simulation (Monte Carlo)

Probabilistic planning involving activity durations

An Illustration



Activity	Optimistic	Most Likely	Pessimistic	Mean	Standard Dev.	Variance
R	3	4	8	4.50	0.833	0.69444
A	3	4	5	4.00	0.333	0.11111
G	2	3	5	3.17	0.500	0.25000
C	3	6	7	5.67	0.667	0.44444
J	2	5	6	4.67	0.667	0.44444
M	1	3	5	3.00	0.667	0.44444
W	1	2	4	2.17	0.500	0.25000

- Calculate the Expected Duration of each path, and Identify the Critical Path on the basis of the mean only:

Activity	Optimistic	Most Likely	Pessimistic	Mean	Standard Dev.	Variance
R	3	4	8	4.50	0.833	0.69444
A	3	4	5	4.00	0.333	0.11111
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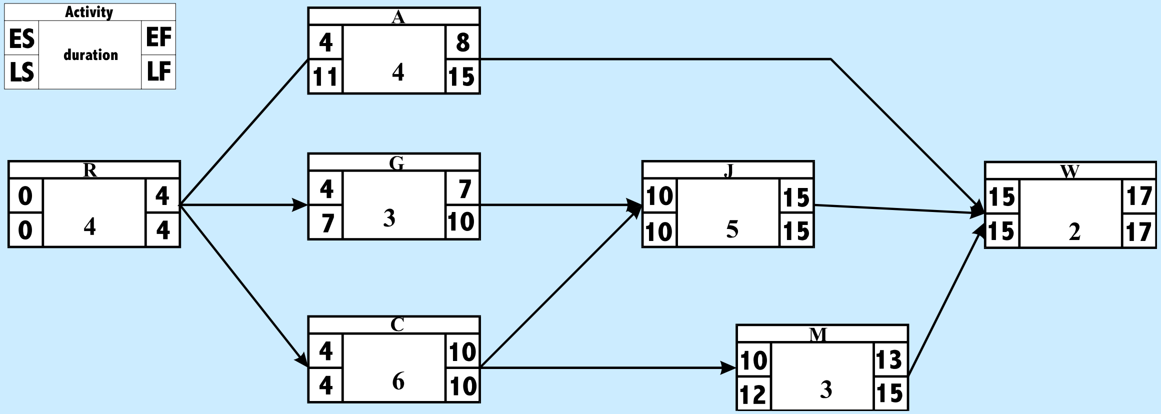
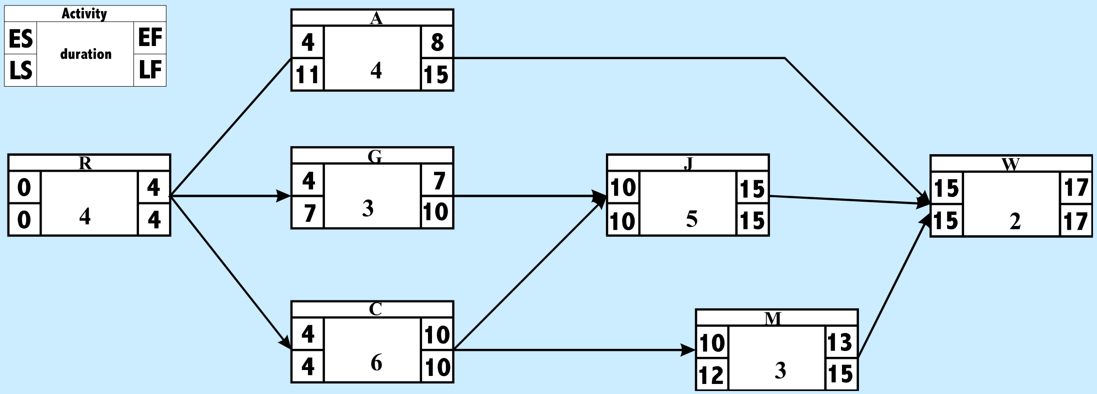
C	3	6	7	5.67	0.667	0.44444
J	2	5	6	4.67	0.667	0.44444
M	1	3	5	3.00	0.667	0.44444
W	1	2	4	2.17	0.500	0.25000

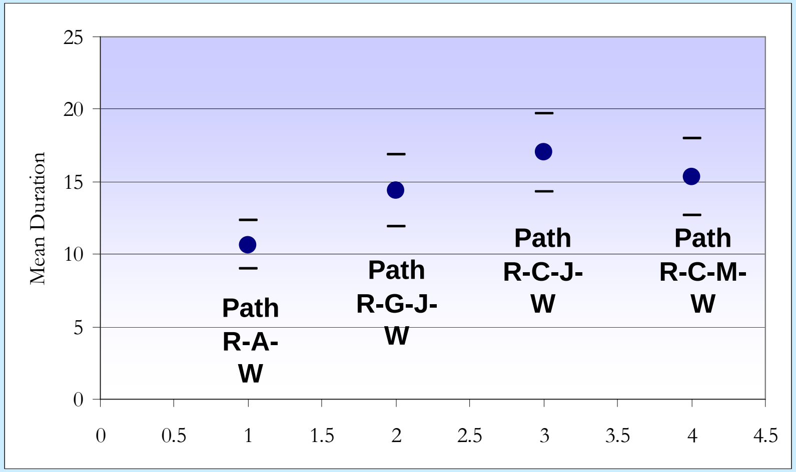


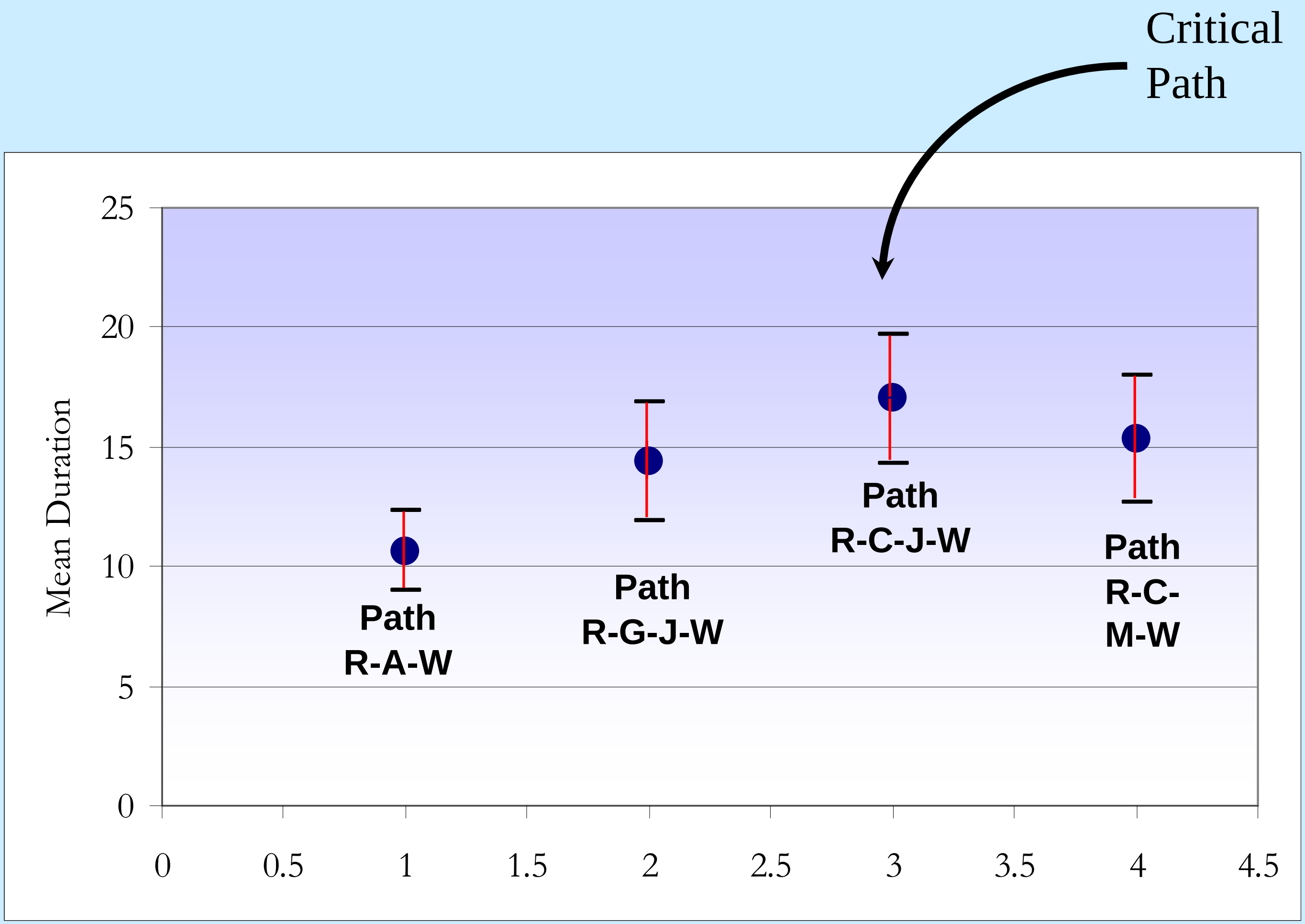
Path:	Deterministic Duration:	Probabilistic Duration:	
R-A-W	10	10.67	
R-G-J-W	14	14.50	
R-C-J-W	17	17.00	***Critical Path***
R-C-M-W	15	15.33	

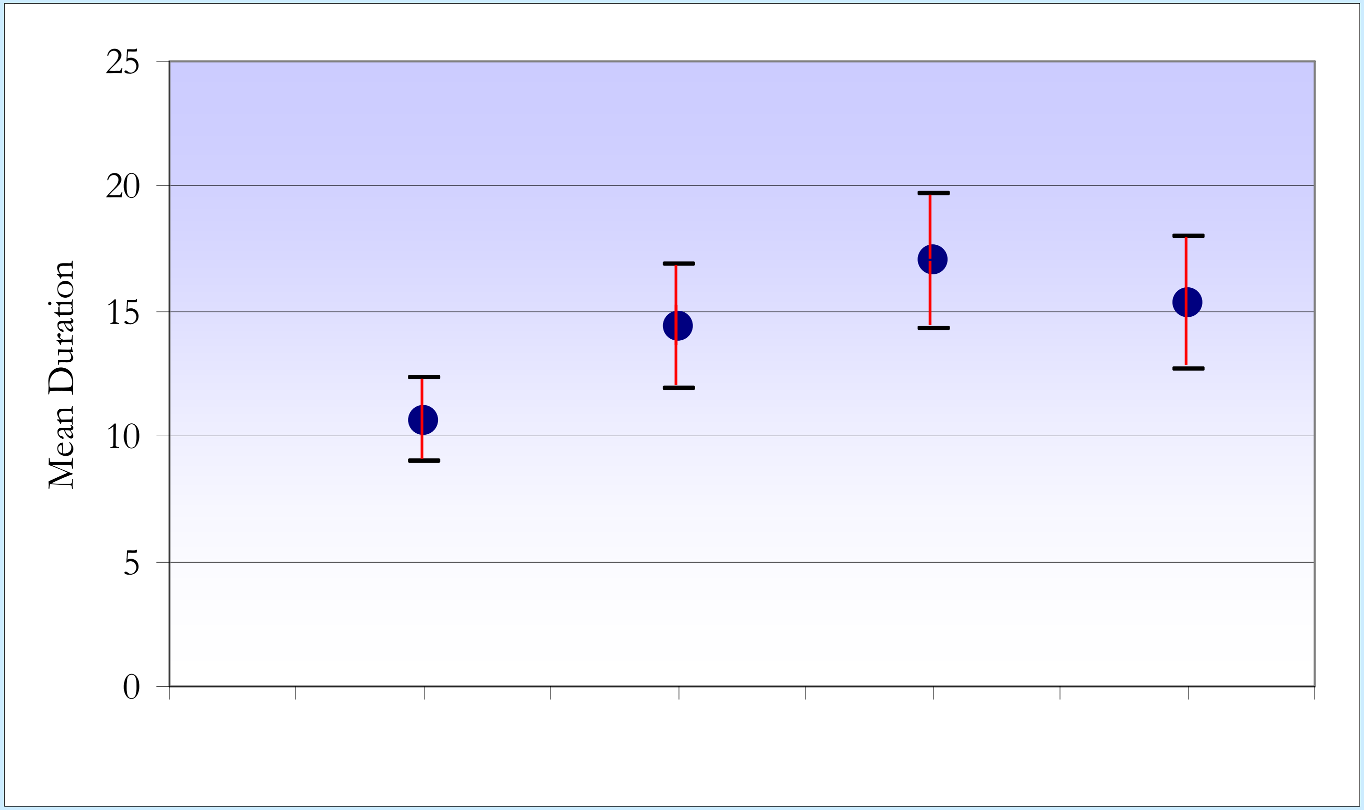
- Calculate the Expected Duration of each path, and Identify the Critical Path <u>on the basis of both the mean and the std dev</u>:

Activity	Optimistic	Most Likely	Pessimistic	Mean	Standard Dev.	Variance
R	3	4	8	4.50	0.833	0.69444
A	3	4	5	4.00	0.333	0.11111
G	2	3	5	3.17	0.500	0.25000
C	3	6	7	5.67	0.667	0.44444
J	2	5	6	4.67	0.667	0.44444
M	1	3	5	3.00	0.667	0.44444
W	1	2	4	2.17	0.500	0.25000





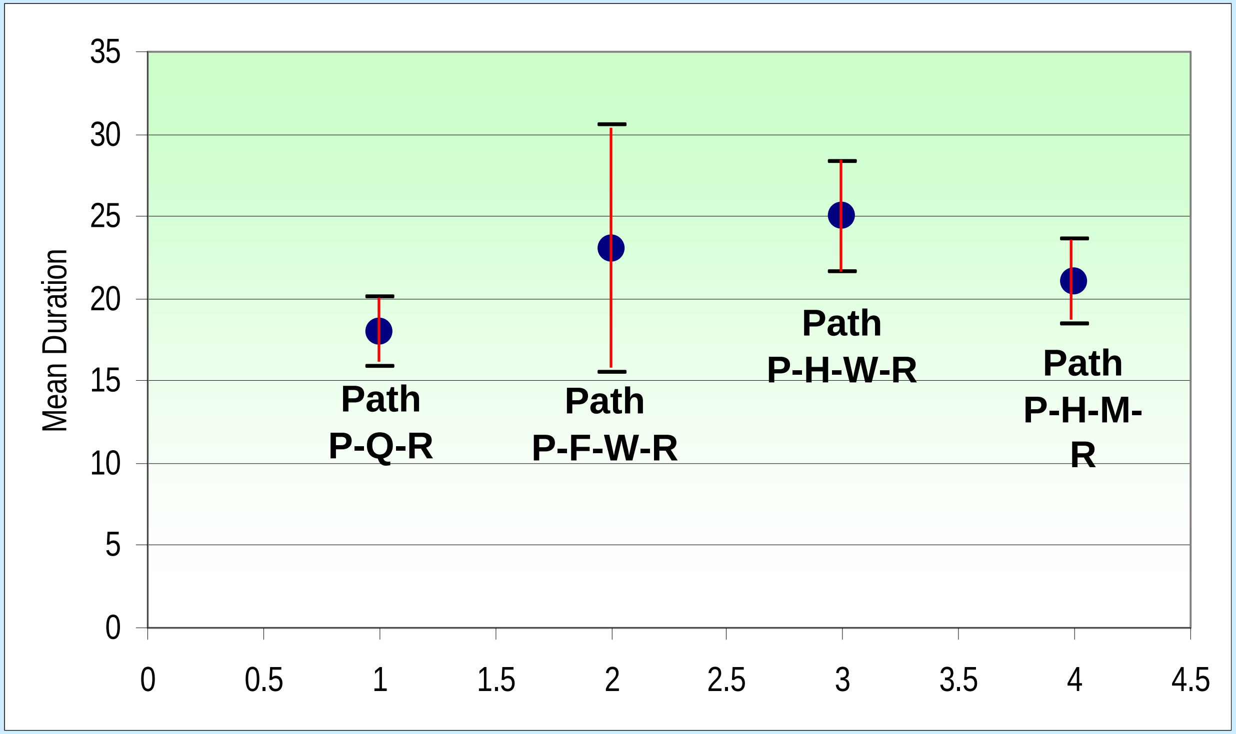

Critical
Path
25
20
15
10
5
0
Mean Duration
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Path
R-A-W
Path
R-G-J-W
Path
R-C-J-W
Path
R-C-
M-W



Critical path here can be considered as that with:

- Longest duration (mean)
- Greatest variation (stdev)

Consider the following hypothetical project paths:



On the basis of <u>mean</u> duration only, Path P-H-W-R is the critical path

On the basis of the <u>variance</u> of durations only, Path P-F-W-R is the critical path

How would you decide the critical path on the basis of both mean duration and variance of durations?

Better way to identify critical path is using the amount of slack in each path (see later slides)

Another Example of Probabilistic Project Scheduling

- Monte Carlo Simulation
- Similar activity structure as before, but Start and End activities are dummies (zero durations).

See Excel Sheet Attached

Benefits of Probabilistic Project Planning

Discussed in previous slides

- Helps identify likely critical paths in situations where there is great uncertainty

- Helps ascertain the likelihood (probability) that overall project duration will fall within a given range

- Helps establish a scale of "criticality" among the project activities

Benefits of Probabilistic Project Planning

- Helps identify likely critical paths in situations where there is great uncertainty
- Helps ascertain the likelihood (probability) that overall project duration will fall within a given range
- Helps establish a scale of "criticality" among the project activities

Discussed in subsequent slides

Probabilistic planning …

Is it ever used in real-life project management?

A Tool for Stochastic Planning: PERT

- Program Evaluation and Review Technique (PERT)

  - Need for PERT arose during the Space Race, in the late fifties

  - Developed by Booz-Allen Hamilton for US Navy, and Lockheed Corporation

    - Polaris Missile/Submarine Project

    - R&D Projects

    - Time Oriented

    - Probabilistic Times

    - Assumes that activity durations are Beta distributed

PERT Parameters

- Optimistic duration a
- Most Likely duration m
- Pessimistic duration b
- Expected duration $\bar{d} = \frac{1}{3}\left[2m + \frac{1}{2}(a+b)\right] = \frac{a+4m+b}{6}$
- Standard deviation $s = \frac{b-a}{6}$
- Variance $v = s^2$

Steps in PERT Analysis

- Obtain a, m and b for each activity
- Compute Expected Activity Duration $d=t_e$
- Compute Variance $v=s^2$
- Compute Expected Project Duration $D=T_e$
- Compute Project Variance $V=S^2$ as Sum of Critical Path Activity Variance
- In Case of Multiple Critical Path Use the One with the Largest Variance
- Calculate Probability of Completing the Project

PERT Example

Activity	Predecessor	a	m	b	d	v
A	-	1	2	4	2.17	0.25
B	-	5	6	7	6.00	0.11
C	-	2	4	5	3.83	0.25
D	A	1	3	4	2.83	0.25
E	C	4	5	7	5.17	0.25
F	A	3	4	5	4.00	0.11
G	B,D,E	1	2	3	2.00	0.11

PERT Example

Finding the Standard Deviation of the duration of a given path comprising Activities C, E, and G.

$$T_e = 11$$

$$S^2 = V[C] + V[E] + V[G]$$

$$= 0.25 + 0.25 + 0.1111$$

$$= 0.6111$$

$$S = \sqrt{0.6111}$$

$$= 0.7817$$

PERT Analysis

Finding probability that project duration is less than some value

Example: probability that project ends before 10 months

$$
\begin{aligned}
P\left(T \le T_d\right) &= P(T \le 10) \\
&= P\left(z \le \frac{10 - T_e}{S}\right) \\
&= P\left(z \le \frac{10 - 11}{0.7817}\right) \\
&= P(z \le -1.2793) \\
&= 1 - P(z \le 1.2793) \\
&= 1 - 0.8997 \\
&= 0.1003 \\
&= 10\%
\end{aligned}
$$

Probability that the project will end before 13 months

$$P(T \leq 13) = P\left(z \leq \frac{13 - 11}{0.7817}\right)$$

$$= P(z \leq 2.5585)$$

$$= 0.9948$$

Probability that the project will have a duration between 9 and 11.5 months

$$
\begin{aligned}
P\left(T_L \le T \le T_U\right) &= P(9 \le T \le 15) \\
&= P(T \le 11.5) - P(T \le 9) \\
&= P\left(z \le \frac{11.5 - 11}{0.7817}\right) - P\left(z \le \frac{9 - 11}{0.7817}\right) \\
&= P(z \le 0.6396) - P(z \le -2.5585) \\
&= P(z \le 0.6396) - \left[1 - P(z \le 2.5585)\right] \\
&= 0.7389 - [1 - 0.9948] \\
&= 0.7389 - 0.0052 \\
&= 0.7337
\end{aligned}
$$

PERT Advantages

- Includes Variance
- Assessment of Probability of Achieving a Goal

PERT Disadvantages

- Data intensive - Very Time Consuming
- Validity of Beta Distribution for Activity Durations
- Only one Critical Path considered
- Assumes independence between activity durations

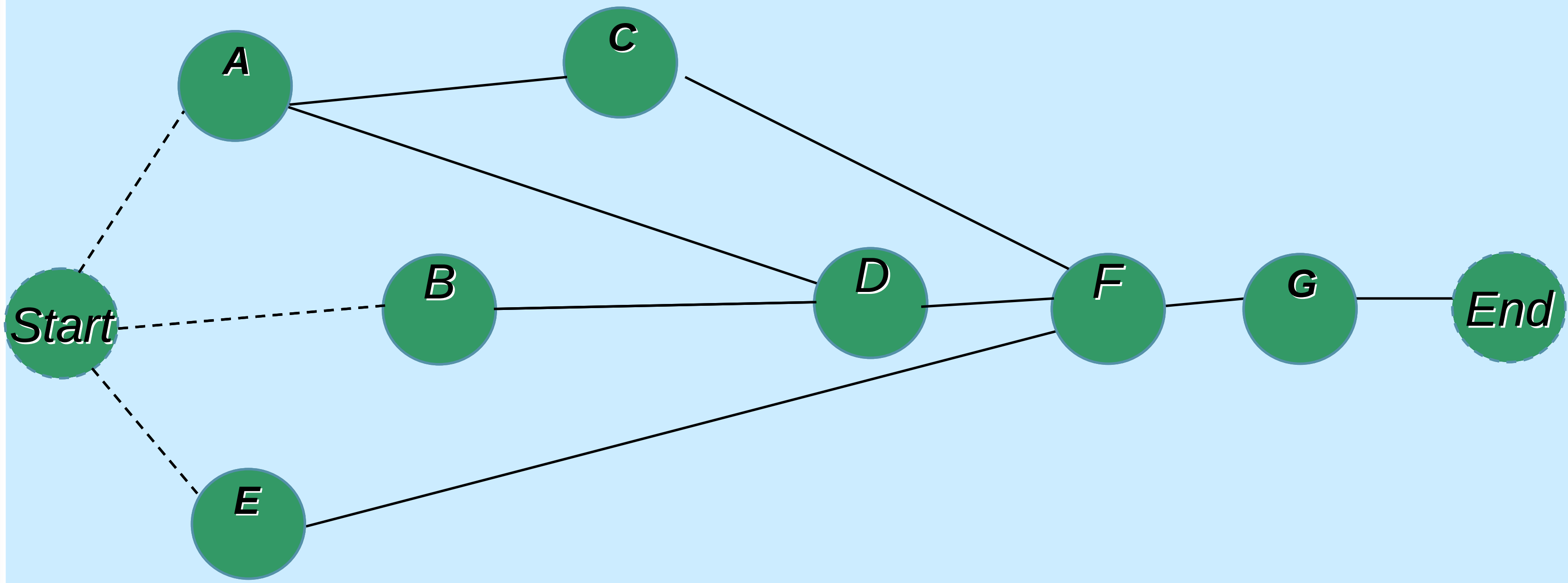
PERT Monte Carlo Simulation

- Determine the Criticality Index of an Activity
- Used 10,000 Simulations, Now from 1000 to 400 Have Been Reported as Giving Good Results

PERT Monte Carlo Simulation Process

- Set the Duration Distribution for Each Activity
- Generate Random Duration from Distribution
- Determine Critical Path and Duration Using CPM
- Record Results

Example Network



Monte Carlo Simulation Example

Statistics for Example Activities

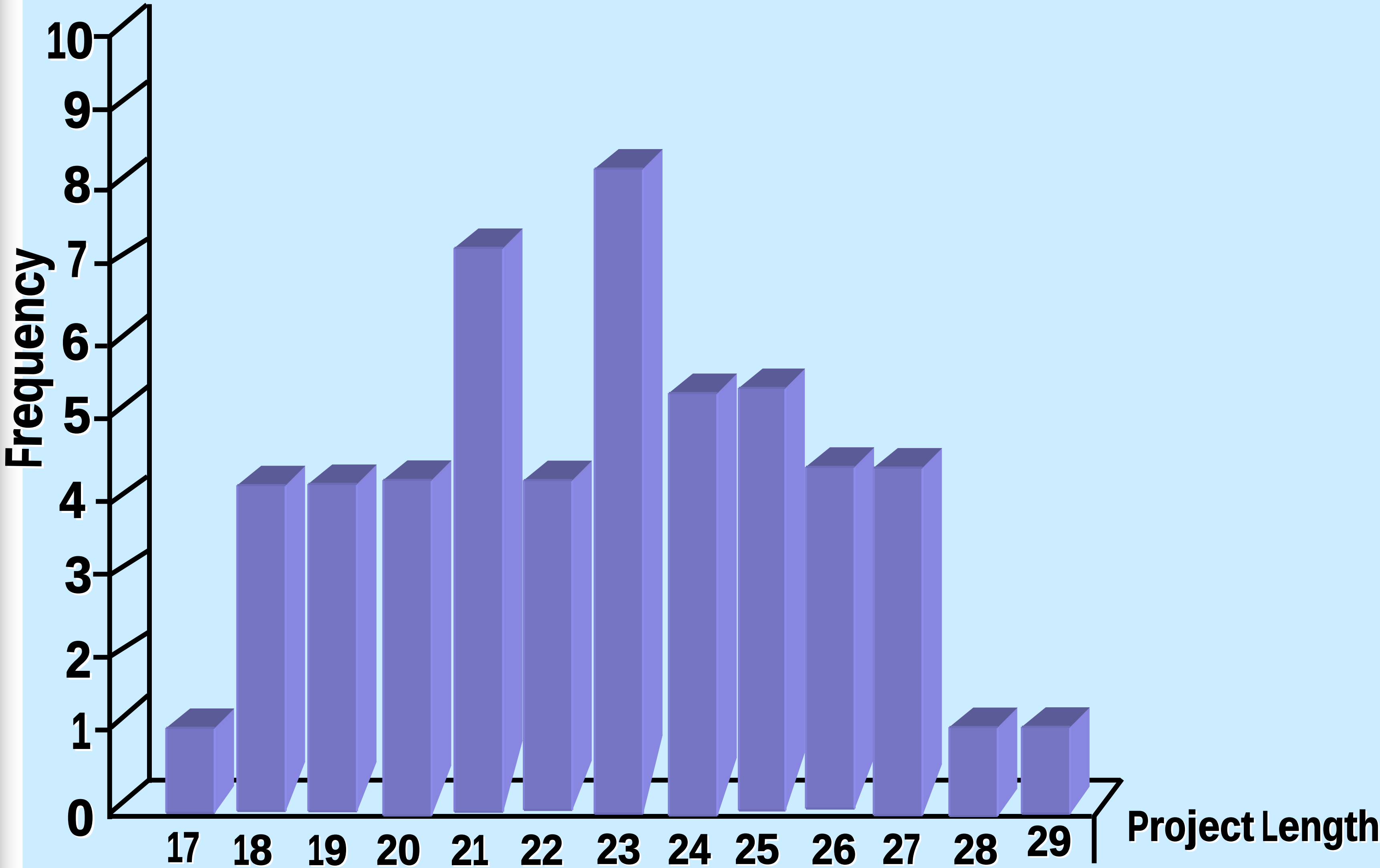
Activity	Optimistic Time, a	Most Likely Time, m	Pessimistic Time, b	Expected Value, $\bar{d}$	Standard Deviation, s
A	2	5	8	5	1
B	1	3	5	3	0.66
C	7	8	9	8	0.33
D	4	7	10	7	1
E	6	7	8	7	0.33
F	2	4	6	4	0.66
G	4	5	6	5	0.33

Monte Carlo Simulation Example

Summary of Simulation Runs for Example Project

				Activity Duration					
Run Number	A	B	C	D	E	F	G	Critical Path	Completion Time
1	6.3	2.2	8.8	6.6	7.6	5.7	4.6	A-C-F-G	25.4
2	2.1	1.8	7.4	8.0	6.6	2.7	4.6	A-D-F-G	17.4
3	7.8	4.9	8.8	7.0	6.7	5.0	4.9	A-C-F-G	26.5
4	5.3	2.3	8.9	9.5	6.2	4.8	5.4	A-D-F-G	25.0
5	4.5	2.6	7.6	7.2	7.2	5.3	5.6	A-C-F-G	23.0
6	7.1	0.4	7.2	5.8	6.1	2.8	5.2	A-C-F-G	22.3
7	5.2	4.7	8.9	6.6	7.3	4.6	5.5	A-C-F-G	24.2
8	6.2	4.4	8.9	4.0	6.7	3.0	4.0	A-C-F-G	22.1
9	2.7	1.1	7.4	5.9	7.9	2.9	5.9	A-C-F-G	18.9
10	4.0	3.6	8.3	4.3	7.1	3.1	4.3	A-C-F-G	19.7

Project Duration Distribution



Probability Computations from Monte Carlo results

$$P(T \le T^*) = \frac{\text{Number of Times Project Finished in Less Than or Equal to } T^* \text{ u}}{\text{Total Number of Replications}}$$

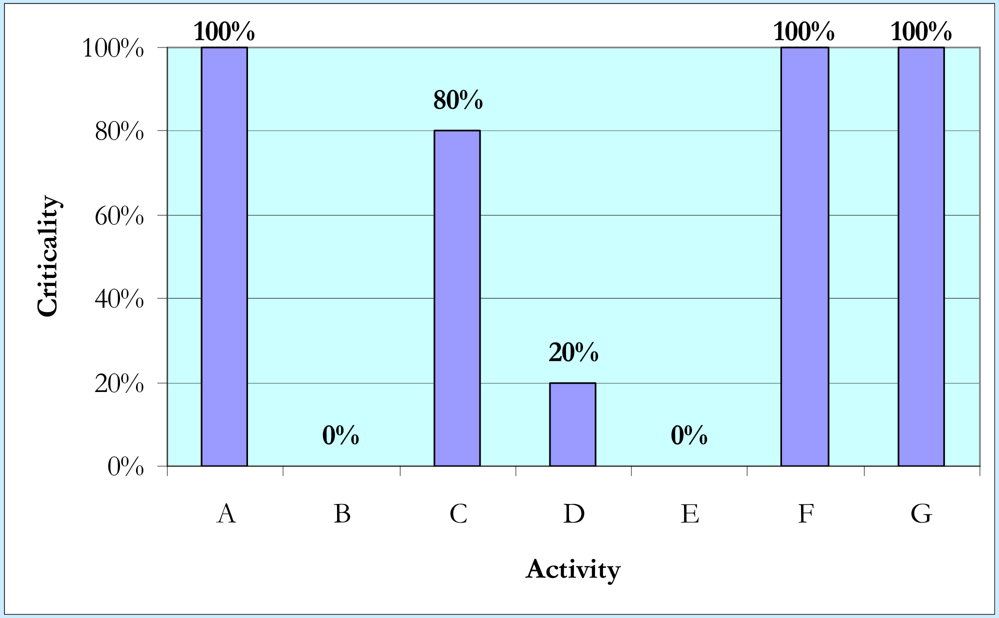
$$P(T \ge T^*) = \frac{\text{Number of Times Project Finished in More Than or Equal to } T^*}{\text{Total Number of Replications}}$$

ETC.

Criticality Index for an Activity

Definition:

Proportion of Runs in which the Activity is in the Critical Path



Criticality Index for a Path

Definition I ("Naïve Definition):

Proportion of Runs in which the Activity is in the Critical Path (see Slide # 60)

Criticality Index for a Path

Definition I ("Naïve Definition):

Proportion of Runs in which the Activity is in the Critical Path (see Slide # 60)

Summary of Simulation Runs for Example Project

				Activity Duration					
Run Number	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**Critical Path**	**Completion Time**
1	6.3	2.2	8.8	6.6	7.6	5.7	4.6	A-C-F-G	25.4
2	2.1	1.8	7.4	8.0	6.6	2.7	4.6	A-D-F-G	17.4
3	7.8	4.9	8.8	7.0	6.7	5.0	4.9	A-C-F-G	26.5
4	5.3	2.3	8.9	9.5	6.2	4.8	5.4	A-D-F-G	25.0
5	4.5	2.6	7.6	7.2	7.2	5.3	5.6	A-C-F-G	23.0
6	7.1	0.4	7.2	5.8	6.1	2.8	5.2	A-C-F-G	22.3
7	5.2	4.7	8.9	6.6	7.3	4.6	5.5	A-C-F-G	24.2
8	6.2	4.4	8.9	4.0	6.7	3.0	4.0	A-C-F-G	22.1
9	2.7	1.1	7.4	5.9	7.9	2.9	5.9	A-C-F-G	18.9
10	4.0	3.6	8.3	4.3	7.1	3.1	4.3	A-C-F-G	19.7

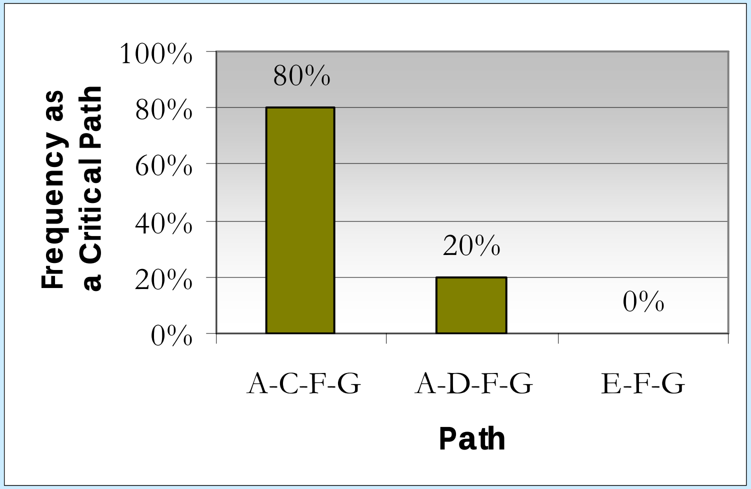
PATH	Frequency as a Critical Path	%
A-C-F-G	8	80%
A-D-F-G	2	20%
E-F-G	0	0%
	10	

Criticality Index for a Path

Definition I:

Proportion of Runs in which the Activity is in the Critical Path (see Slide # 60)

PATH	Frequency as a Critical Path	%
A-C-F-G	8	80%
A-D-F-G	2	20%
E-F-G	0	0%
	10	



Criticality Index for a Path

Definition II:

How much slack exists in that path.

Less Slack → Higher criticality

More Slack → Lower criticality

$$\lambda = \frac{\alpha_{max} - \beta}{\alpha_{max} - \alpha_{min}}(\mathbf{100\%})$$

Ranges from 0% to 100%

α_{min} = minimum total float

α_{max} = maximum total float

β = total float or slack in current path

Using the index, we can rank project paths from most critical to least critical

Criticality Index for a Path

Definition II:

See Example In Excel File

(Path Criticality Slide)

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