Answer 5.2.

a) and b) For all cases the length scale is L = 100 m and the diffusion rate $D = 1 m^2 s^{-1}$.

U [m s-1]	Diffusion: $T_D = L^2/8D$	Advection: $T_U = L/U$	Pe = UL/D	Curve
0.001	1250 s	100,000 s	0.1	Blue
0.1	1250 s	1,000 s	10	Green
1	1250 s	100 s	100	Red

c) For Fickian diffusion, the peak concentration occurs at the center of mass which arrives at the advection time scale, i.e. at time T_U the peak concentration in the cloud will be located at L. However, the peak concentration observed at L will not necessarily occur at T_U. This is because the magnitude of the concentration is changing as the cloud passes the measurement point. If the cloud is passing very slowly (Pe << 1), the concentration may decline considerably as the cloud is passing, such that the peak concentration seen at L will occur before the center of mass arrives. If the concentration changes very little as the cloud passes (advection is faster than diffusion, Pe >>1) the peak concentration observed at L will correspond to the advection time scale. We expect this to occur for the Red and possibly the Green systems.

To find the concentration we must first determine whether the cloud is three-, two-, or one-dimensional as it reaches the measurement position. This will depend on the time required to mix the dye across the channel area, $A = 10 m^2$. If we assume a square cross-section, then the width and depth of the channel is, $L_y = L_z = \sqrt{10}$ m. The mixing time-scale (see chap 4, eq 24), is $t_i = L_i^2 / 4D_i = 10m^2/(4 m^2 s^{-1}) = 2.5$ s. For each curve both T_D and T_U >> 2.5 s, so we can safely assume that the dye is well-mixed across the channel when it arrives at L. Thus, we can use a one-dimensional solution to estimate the concentration observed at L at t = L/U.

Red Curve: $$C(x = L, t = L/U) = \frac{M}{A\sqrt{4\pi Dt}} = \frac{1000g}{10m^2\sqrt{4\pi(1m^2s^{-1})(100s)}} = 2.82\ gm^{-3}.$$

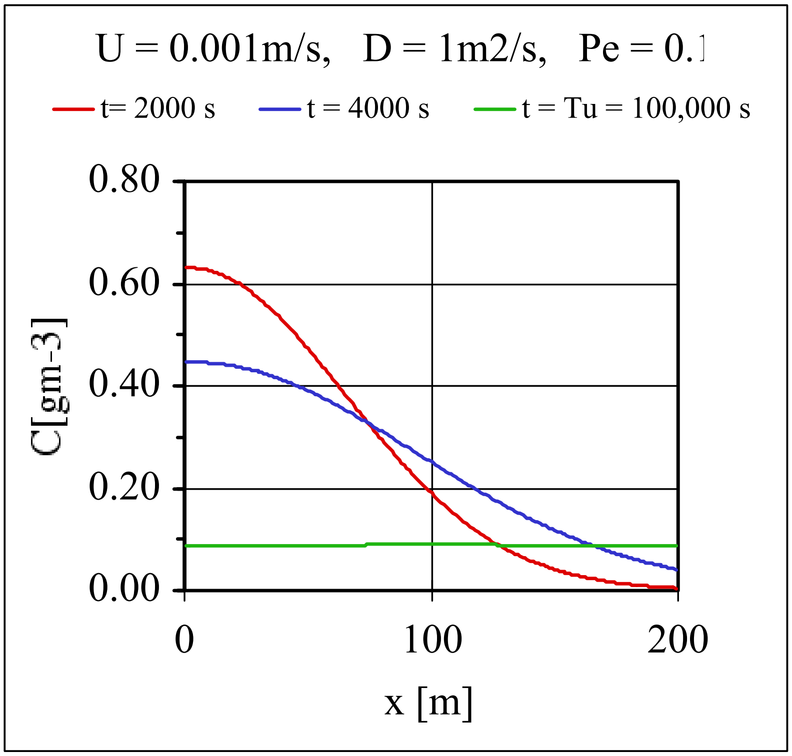
This value is consistent with the peak concentration observed at L.

For the Green Curve $$C(x=L, t=L/U) = \frac{1000g}{10m^2\sqrt{4\pi(1m^2s^{-1})(1000s)}} = 0.89\ gm^{-3}$$

This value is also consistent with the peak observed concentration.

For the Blue Curve $$C(x=L, t=L/U) = \frac{1000g}{10m^2\sqrt{4\pi(1m^2s^{-1})(100,000s)}} = 0.09\ gm^{-3}$$

This is LESS than the concentration observed at L at t = 4000s ($C = 0.25\ gm^{-3}$). For this case the peak concentration observed at L occurs before T_U, i.e. before the center of mass passes L. This is because the concentration in the cloud is dropping off faster (via diffusion) than the center of mass can travel (via advection), consistent with Pe << 1. The spatial distribution, C(x), for this condition is shown below.



d) We are interested in the concentration observed at L = 100m. In all cases the transport time scale for this location is 100 s or more, which is much longer than the injection time scale. For this reason the injection can be assumed to be instantaneous.