

Answer 4.1 - Describe the concentration field inside the box from $t = 0$ to $t = L^2/D$.

Hint 1 - When will the mass be mixed uniformly in the vertical?

From equation 4.24 a mass released mid-way between two parallel boundaries a distance L_z apart will be mixed to a uniform concentration between those boundaries in time $t = L_z^2/4D$, where L_z is the distance between the boundaries. The time for the concentration to become well-mixed in the vertical is then, $t = (0.01L)^2/D = 0.0001 L^2/D$.

Hint 2 - Estimate when the mass will reach each vertical wall in the box.

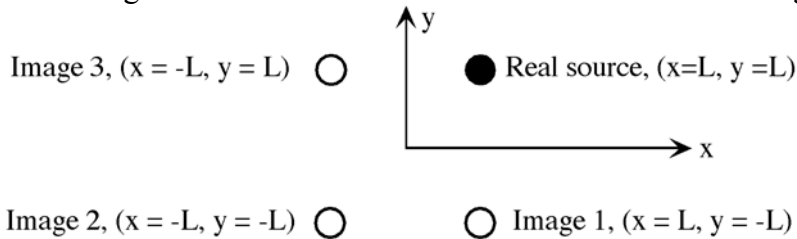
We estimate the time that the cloud will first touch a boundary based on the standard deviation of the mass distribution. The cloud will touch the top and bottom walls when, $3\sigma = 3\sqrt{2Dt} = L$, or at the time $t = L^2/18D$. The cloud will touch the far wall when $3\sigma = 3\sqrt{2Dt} = 99L$, or at the time $t \approx 545L^2/D$.

Hint 3 - How will each boundary impact the solution in the time $t = 0$ to L^2/D ?

Use the time scales determined in hint 1 and 2. Because the concentration field becomes uniform in the vertical very rapidly, within $1/10,000^{\text{th}}$ of the time of interest, we will assume that the concentration is instantly uniform in z , i.e. $\partial C/\partial z = 0$ for all time. In the time of interest, the cloud will never reach the far walls ($545L^2/D \gg L^2/D$), so these boundaries do not impact the solution. The cloud will reach the near walls ($L^2/18D < L^2/D$), and a correction must be made to satisfy the no flux condition at these walls.

Hint 4 - Place images sources to satisfy the no-flux boundary condition.

Image sources are needed at the following locations. Image 2 balances the loss of mass from Image 3 across the x -axis and the loss of mass from Image 1 across the y -axis.



Solution - The concentration in the box is described by a superposition of two-dimensional, instantaneous, slug releases (equation 3.23) at each of the above sources.

$$C(x, y, t) = \frac{M}{L_z 4\pi Dt} \cdot \left(\exp\left(-\frac{(x-L)^2 + (y-L)^2}{4Dt}\right) + \exp\left(-\frac{(x-L)^2 + (y+L)^2}{4Dt}\right) + \exp\left(-\frac{(x+L)^2 + (y+L)^2}{4Dt}\right) + \exp\left(-\frac{(x+L)^2 + (y-L)^2}{4Dt}\right) \right)$$

real source
image 1
image 2
image3