Answer 5.5.

a. Assume the flow fills the channel uniformly, then $U = Q/A = 2.5 \times 10^{-4}\ ms^{-1}$. Using L = 75, Pe = 0.19. Alternatively, for L = (75-25) = 50 m, Pe = 0.13. In either case the Peclet number indicates that the system is dominated by diffusion.

b. According to the Peclet number, transport is dictated by diffusion. The time-scale for the contaminant to reach the harbor can be estimated as the time-scale for diffusive transport over the 50-m between the spill and the harbor entrance. $T_D = (50m)^2/(8 \times 0.1\ m^2s^{-1}) = 3125\ s \approx 1hr$. So, I have one hour to put up a contaminant-absorbing barrier and protect the harbor

c. In the vertical, $t_{mix} = (2\ m)^2/(4 \times 0.1\ m^2s^{-1}) = 10\ s$
In the lateral, $t_{mix} = (10\ m)^2/(4 \times 0.1\ m^2s^{-1}) = 250\ s$

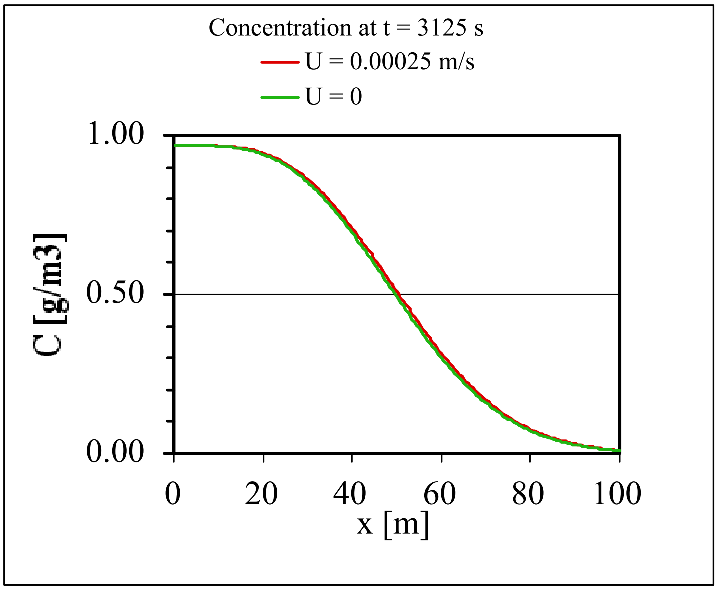
d. Since the mixing time scales in both the lateral and vertical are much shorter than the time required for the contaminant to reach the harbor, we can assume the contaminant is mixed across the channel area when it reaches x = 75 m. If concentration is uniform (well-mixed) in the lateral and vertical, we can drop these two dimensions, and use a one-dimensional solution.

e.
$$C(x,t) = \frac{M}{A_{yz}\sqrt{4\pi Dt}}\left[\underbrace{\exp\left(-\frac{(x-25-Ut)^2}{4Dt}\right)}_{\text{real source}} + \underbrace{\exp\left(-\frac{(x+25-Ut)^2}{4Dt}\right)}_{\text{image source}}\right]$$

The image source is needed to satisfy the no-flux boundary at x = 0. Since Pe<<1, we could also neglect U entirely and still get a good representation of C(x,t).

$$C(x,t) = \frac{M}{A_{yz}\sqrt{4\pi Dt}}\left[\exp\left(-\frac{(x-25)^2}{4Dt}\right) + \exp\left(-\frac{(x+25)^2}{4Dt}\right)\right]$$

A comparison of the full solution (U = 0.00025 m/s) and the solution that neglects U is given below. One can quickly see that U is indeed negligible, as implied by Pe.



f. U and D are assumed to be uniform, i.e. not functions of (x, y, z), but in fact the no-slip condition at the channel boundaries will make U = f(y,z).
We assume that D is isotropic. In fact, turbulent diffusion in the longtudinal direction will be much more rapid than in the vertical or horizontal. This is discussed further in Chapter 9. We neglect losses to the atmosphere, when in fact the gasoline is volatile. We assume that no gasoline absorbs to the sediment. We assume that the flow and thus velocity are not functions of time.