

Solution to Problem 1.3

Question 1

t (day)	t (sec)	C_{\max}	$0.6 C_{\max}$	σ (m)
1	86400	13.5	8.1	3.0
4	345600	6.8	4.1	6.0

Assuming you released the dye in a very thin horizontal layer, such that initially $\sigma = 0$, then:

$$\sigma = \sqrt{2Dt} \Rightarrow D = \frac{\sigma^2}{2t}$$

Thus, our two estimates of D are:

$$D_1 = \frac{(3.0 \text{ m})^2}{2(86400 \text{ s})} = 5.2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}, \text{ and}$$

$$D_2 = \frac{(6.0 \text{ m})^2}{2(345600 \text{ s})} = 5.2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}.$$

Therefore, the estimated diffusion coefficient of the dye is $5.2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$.

Question 2

We must consider the boundaries of the tank when the width of the cloud (4σ) is 40 m.

That is, when:

$$4\sqrt{2Dt} = 40 \Rightarrow Dt = 50 \text{ m}^2.$$

Using our value of D above, the boundaries of the tank become important when

$$t = \frac{50 \text{ m}^2}{5.2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}} = 9.6 \times 10^5 \text{ s} = \mathbf{11.1 \text{ days}}.$$