Answer 5.1.

First, determine if the flow is laminar. The velocity in the tube is U = Q/A = 0.1 cm/s. The diameter of the tube is d = $\sqrt{(4/\pi)A}$ = 1.13 cm. The Reynolds number is Re = UD/ν = (0.1 cm/s)(1.13 cm)/(0.01 cm^2s^{-1}) = 11.3 << 2100, the limit for laminar flow.

Here, Pe = UL/D = (0.1 cm/s)(100 cm)/(10^{-5} cm^2s^{-1}) = 10^6. Since Pe >>1 the arrival of the dye at L can be estimated by L/U = 1000 s. At this time the dye patch will have a length, $4\sigma = 4\sqrt{2Dt} = 4\sqrt{2(10^{-5}\,cm^2s^{-1})(1000s)}$ = 0.57 cm. The maximum concentration will occur when the center of mass passes that point, i.e. at t = L/U. So, the maximum concentration will be C (x = L, t = L/U = 1000s). Since the mass mixes instantly across the cross-section, and the flow is uniform across the section, the system is effectively one dimensional. The concentration resulting from an instantaneous point release is

$$C(x,t) = \frac{M}{A_{yz}\sqrt{4\pi Dt}}\exp(-(x-ut)^2/4Dt).$$

Evaluated at x = L and t = L/U, we have C (x =L, t =L/U) =

$$\frac{M}{A\sqrt{4\pi DL/U}} = \frac{1\text{ g}}{1cm^2\sqrt{4\pi(10^{-5}\,cm^2s^{-1})(100\text{ cm})/(0.1\text{ cm/s})}} = 28.2\text{ g cm}^{-3}$$