Urban OR Problem Set 5

Due In Class Wednesday, November 17

Also this problem set is more difficult then the others and it is recommended that you start on it early as the concepts here combine everything else you have learned thus far.

Problem #1. Two Server Hypercube Problem

Consider a 1-mile-by-2-mile homogeneous service region served by two mobile patrolling servers as shown in Figure 1. Here are the assumptions of the model:

1. Customer locations are uniformly independently located over the entire rectangular service region.
2. Over time, customers arrive as a homogeneous Poisson process at aggregate rate λ=2 customers per hour.
3. While not busy serving customers, each server patrols its sector (sector 1 or 2, respectively). Under this circumstance, the server's location at a random time is uniformly distributed over its sector. Each sector is a one-mile-by-one-mile square.
4. Travel distance is right-angle ("Manhattan metric"), with speed equal to 1000 mi./hr.
5. The on-scene time to serve a customer is a random variable having a negative exponential probability density function with mean equal to 30 minutes. Upon completion of service of a customer at the scene, the server resumes random patrol of his/her sector.
6. This is the dispatch strategy: For a customer from sector i (i=1,2), dispatch server i if available. Else dispatch the other server if available; Else the customer is lost forever.

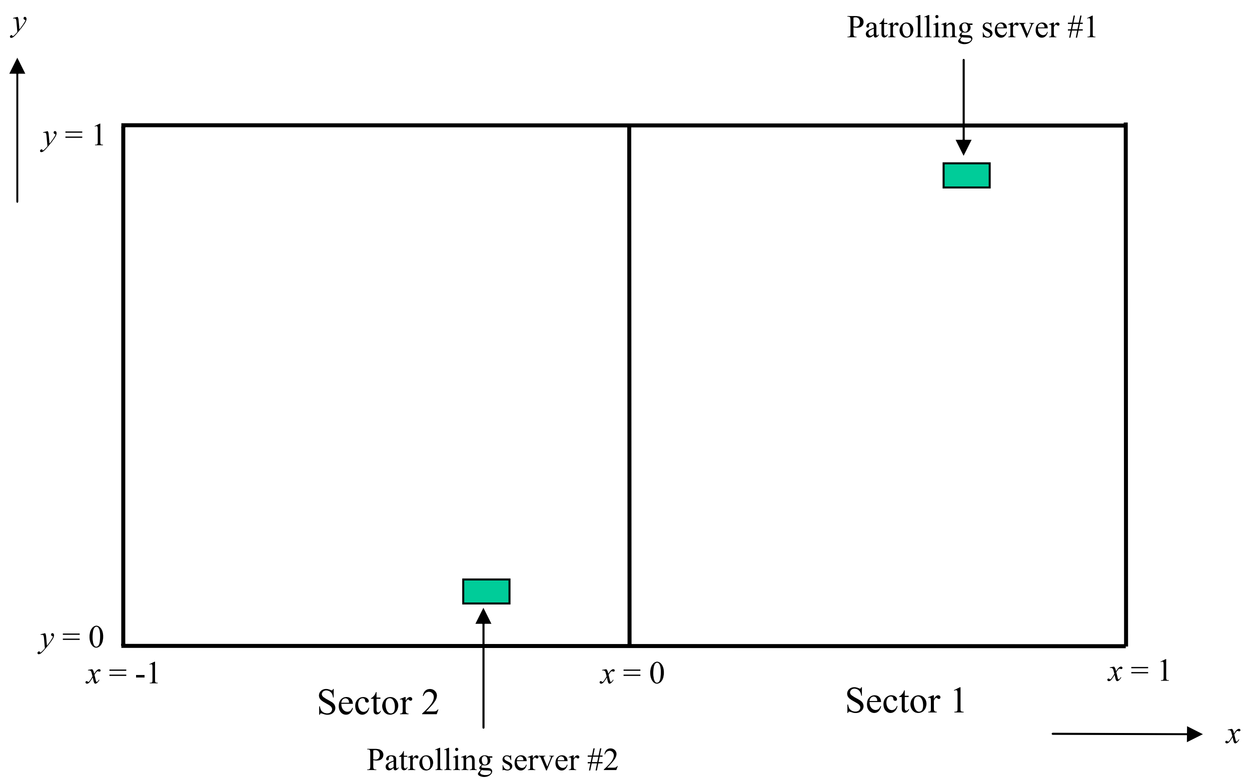

Figure 1

(a) Is it true that the workload (fraction of time busy) of each server is equal to 1/2? If true, briefly explain why. If false, derive the correct figure.
(b) Determine the fraction of dispatches that take server 1 to sector 2.
(c) Determine the mean travel time to a random served customer.

Now consider the situation as shown in Figure 2. Assumptions 1 through 5 above remain correct. However, Assumption 6 is altered as follows:

Case in which both servers are available: For a customer from a part of sector i not in Buffer Zone i, dispatch server i. For a customer in Buffer Zone i, dispatch the other server only if that other server is within its own buffer zone and server i is not within its buffer zone; else dispatch server i to that customer.

Case in which only one server is available: Dispatch that server.

Else the customer is lost forever.

(d) Under this new dispatch policy, determine the fraction of dispatch assignments that send server 1 to sector 2.
(e) Without doing the detailed calculations, describe briefly how you would compute the mean travel time. How would the magnitude of the numerical answer compare to that of part (c)?
(f) Suppose under the simpler dispatch policy #1 above, we find that the workload of Sector #1 is twice the workload of Sector 2, while λ remains the same at $\lambda = 2$. Without doing the calculations, briefly explain how you would find an optimal boundary line separating Sectors 1

and 2, where 'optimal' means minimizing mean travel time. Would it be to the left of right of $x = 0$? Why?

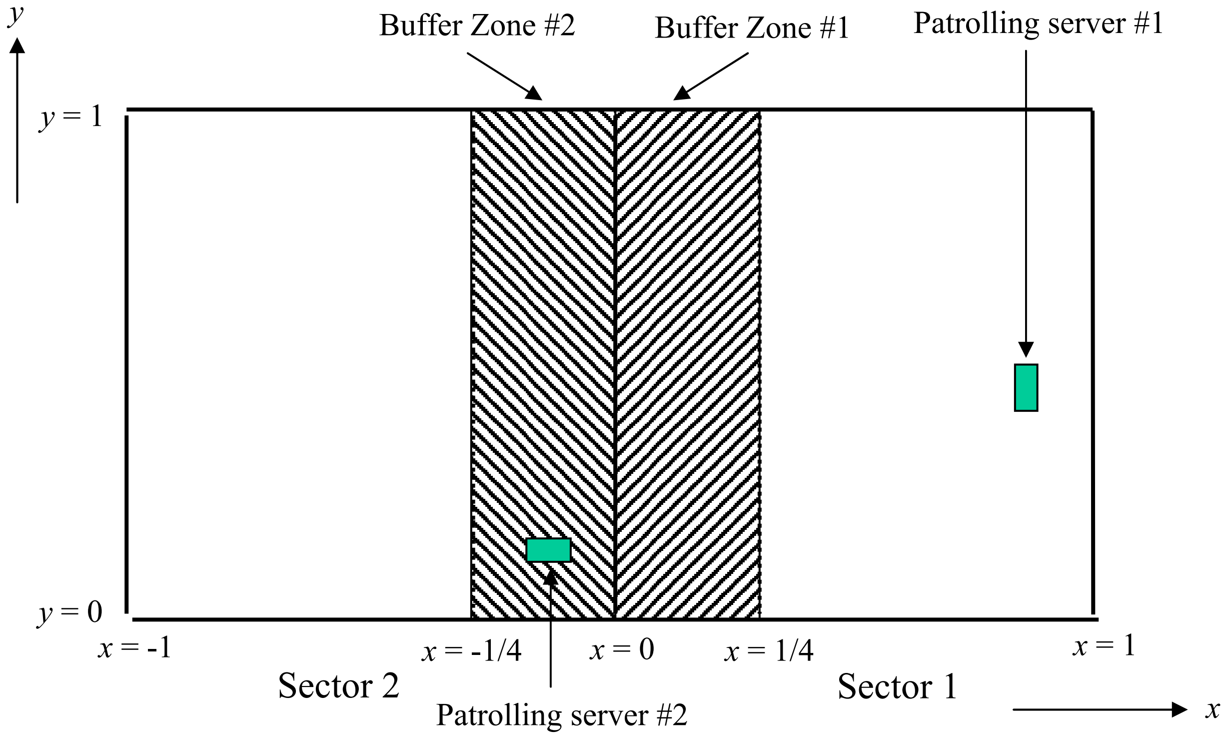

Figure 2

Problem 2: 5.2:

In the part about practical significance please try to think of a real word example where each case would arrive and explain why it is better than the other alternatives.

Problem 4: 5.6