6.012 Electronic Devices and Circuits
Formula Sheet for Hour Exam 1, Fall 2003

Parameter Values:

$q = 1.6x10^{-19}\ Coul$
$\varepsilon_o = 8.854x10^{-14}\ F/cm$
$\varepsilon_{r,Si} = 11.7, \quad \varepsilon_{Si} \approx 10^{-12}\ F/cm$
$n_i[Si@R.T] \approx 10^{10}\ cm^{-3}$
$kT/q \approx 0.025\ V; \quad (kT/q)\ln 10 \approx 0.06\ V$
$1\ \mu m = 1x10^{-4}\ cm$

Periodic Table:

III	VI	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb

Drift/Diffusion:

Drift velocity: $\bar{s}_x = \pm\mu_m E_x$

Conductivity: $\sigma = q(\mu_e n + \mu_h p)$

Diffusion flux: $F_m = -D_m \dfrac{\partial C_m}{\partial x}$

Einstein relation: $\dfrac{D_m}{\mu_m} = \dfrac{kT}{q}$

Electrostatics:

$$\varepsilon \frac{dE(x)}{dx} = \rho(x) \qquad E(x) = \frac{1}{\varepsilon}\int \rho(x)dx$$
$$-\frac{d\phi(x)}{dx} = E(x) \qquad \phi(x) = -\int E(x)dx$$
$$-\varepsilon\frac{d^2\phi(x)}{dx^2} = \rho(x) \qquad \phi(x) = -\frac{1}{\varepsilon}\iint \rho(x)dxdx$$

The Five Basic Equations:

Electron concentration: $\dfrac{\partial n(x,t)}{\partial t} - \dfrac{1}{q}\dfrac{\partial J_e(x,t)}{\partial x} = g_L(x,t) - [n(x,t) \cdot p(x,t) - n_i^2]r(T)$

Hole concentraton: $\dfrac{\partial p(x,t)}{\partial t} + \dfrac{1}{q}\dfrac{\partial J_h(x,t)}{\partial x} = g_L(x,t) - [n(x,t) \cdot p(x,t) - n_i^2]r(T)$

Electron current density: $J_e(x,t) = q\mu_e n(x,t)E(x,t) + qD_e \dfrac{\partial n(x,t)}{\partial x}$

Hole current density: $J_h(x,t) = q\mu_h p(x,t)E(x,t) - qD_h \dfrac{\partial p(x,t)}{\partial x}$

Poisson's equation: $\dfrac{\partial E(x,t)}{\partial x} = \dfrac{q}{\varepsilon}[p(x,t) - n(x,t) + N_d^+(x) - N_a^-(x)]$

Uniform doping, full ionization, TE

n - type, $N_d >> N_a$

$$n_o \approx N_d - N_a \equiv N_D, \quad p_o = n_i^2/n_o, \quad \phi_n = \frac{kT}{q}\ln\frac{N_D}{n_i}$$

p - type, $N_a >> N_d$

$$p_o \approx N_a - N_d \equiv N_A, \quad n_o = n_i^2/p_o, \quad \phi_p = -\frac{kT}{q}\ln\frac{N_A}{n_i}$$

Uniform optical excitation, uniform doping

$$n = n_o + n' \qquad p = p_o + p' \qquad n' = p' \qquad \frac{dn'}{dt} = g_l(t) - (p_o + n_o + n')n'r$$

Low level injection, $n', p' << p_o + n_o$: $\dfrac{dn'}{dt} + \dfrac{n'}{\tau_{min}} = g_l(t)$ with $\tau_{min} \approx (p_o r)^{-1}$

Flow problems (uniformly doped quasineutral regions with quasi-static excitation and low level injection; p-type example):

Minority carrier excess: $$\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = -\frac{1}{D_e} g_L(x) \qquad L_e \equiv \sqrt{D_e \tau_e}$$

Minority carrier current density: $$J_e(x) \approx qD_e \frac{dn'(t)}{dx}$$

Majority carrier current density: $$J_h(x) = J_{Tot} - J_e(x)$$

Electric field: $$E_x(x) \approx \frac{1}{q\mu_h p_o}\left[J_h(x) - \frac{D_h}{D_e} J_e(x)\right]$$

Majority carrier excess: $$p'(x) \approx n'(x) + \frac{\varepsilon}{q}\frac{dE_x(x)}{dx}$$

Non-uniformly doped semiconductor sample in thermal equilibrium

$$\frac{d^2\phi(x)}{dx^2} = \frac{q}{\varepsilon}\left\{n_i\left[e^{q\phi(x)/kT} - e^{-q\phi(x)/kT}\right] - \left[N_d(x) - N_a(x)\right]\right\}$$

$$n_o(x) = n_i e^{q\phi(x)/kT}, \quad p_o(x) = n_i e^{-q\phi(x)/kT}, \quad p_o(x)n_o(x) = n_i^2$$

Depletion approximation for abrupt p-n junction:

$$\rho(x) = \begin{cases} 0 & \text{for} \quad x < -x_p \\ -qN_{Ap} & \text{for} \quad -x_p < x < 0 \\ qN_{Dn} & \text{for} \quad 0 < x < x_n \\ 0 & \text{for} \quad x_n < x \end{cases}$$

$$N_{Ap}x_p = N_{Dn}x_n$$

$$\phi_b \equiv \phi_n - \phi_p = \frac{kT}{q}\ln\frac{N_{Dn}N_{Ap}}{n_i^2}$$

$$w(v_{AB}) = \sqrt{\frac{2\varepsilon_{Si}\left(\phi_b - v_{AB}\right)}{q}\frac{\left(N_{Ap} + N_{Dn}\right)}{N_{Ap}N_{Dn}}}$$

$$\left|E_{pk}\right| = \sqrt{\frac{2q\left(\phi_b - v_{AB}\right)}{\varepsilon_{Si}}\frac{N_{Ap}N_{Dn}}{\left(N_{Ap} + N_{Dn}\right)}}$$

$$q_{DP}(v_{AB}) = -AqN_{Ap}x_p\left(v_{AB}\right) = -A\sqrt{2q\varepsilon_{Si}\left(\phi_b - v_{AB}\right)\frac{N_{Ap}N_{Dn}}{\left(N_{Ap} + N_{Dn}\right)}}$$

Ideal p-n junction diode i-v relation:

$$n(-x_p) = \frac{n_i^2}{N_{Ap}}e^{qv_{AB}/kT}, \quad n'(-x_p) = \frac{n_i^2}{N_{Ap}}\left(e^{qv_{AB}/kT} - 1\right); \qquad p(x_n) = \frac{n_i^2}{N_{Dn}}e^{qv_{AB}/kT}, \quad p'(x_n) = \frac{n_i^2}{N_{Dn}}\left(e^{qv_{AB}/kT} - 1\right)$$

$$i_D = Aqn_i^2\left[\frac{D_h}{N_{Dn}w_{n,eff}} + \frac{D_e}{N_{Ap}w_{p,eff}}\right]\left[e^{qv_{AB}/kT} - 1\right]$$

$$w_{m,eff} = \begin{cases} w_m - x_m & \text{if} \quad L_m \gg w_m \\ L_m \tanh\left[\left(w_m - x_m\right)/L_m\right] & \text{if} \quad L_m \sim w_m \\ L_m & \text{if} \quad L_m \ll w_m \end{cases}$$

$$q_{QNR,p\text{-}side} = Aq\int_{-w_p}^{-x_p} n'(x)dx, \qquad q_{QNR,n\text{-}side} = Aq\int_{x_n}^{w_n} p'(x)dx, \qquad \text{Note: } p'(x) \approx n'(x) \text{ in QNRs}$$