

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Physics

Physics 8.04

Spring 2006

**EXAM 1**

**Tuesday, March 14, 2006**

11:00am-12:30pm

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FAMILY (Last) NAME

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GIVEN (First) NAME

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Student ID Number

**Instructions:**

1. SHOW ALL WORK. All work must be done in the exam booklet.
2. This is a closed book exam.
3. BOOKS, NOTES, COMPUTERS and CELL PHONES are NOT ALLOWED.
4. Do all FOUR (4) problems.
5. You have 90 minutes to solve the problems. Exams will be collected at 12:30pm.

Problem	Maximum	Score	Grader
1	30		
2	30		
3	20		
4	20		
<b>TOTAL</b>	100		

**A. Physical constants and conversion factors**

Quantity	Symbol	Value	Units
Atomic mass unit	amu	931.5	MeV/c <sup>2</sup>
		$1.661 \times 10^{-27}$	kg
Electron mass	m <sub>e</sub>	0.511	MeV/c <sup>2</sup>
		$9.109 \times 10^{-31}$	kg
Proton mass	m <sub>p</sub>	939	MeV/c <sup>2</sup>
		$1.673 \times 10^{-27}$	kg
Elementary charge	q	$1.602 \times 10^{-19}$	C
Planck's constant	h	$6.626 \times 10^{-34}$	J·s
		$4.136 \times 10^{-15}$	eV·s
	$\hbar = h/2\pi$	$1.054 \times 10^{-34}$	J·s
		$6.583 \times 10^{-16}$	eV·s
Speed of light in vacuum	c	$2.998 \times 10^8$	m/s
Boltzmann's constant	k <sub>B</sub>	$1.381 \times 10^{-23}$	J/K
Permittivity of free space	ε <sub>0</sub>	$8.854 \times 10^{-12}$	C <sup>2</sup> N <sup>-1</sup> m <sup>-2</sup>
Avogadro's number	N <sub>A</sub>	$6.022 \times 10^{23}$	mol <sup>-1</sup>
Rydberg constant	R <sub>∞</sub>	$3.158 \times 10^{15}$	Hz

$$1\text{eV} = 1.602 \times 10^{-19} \text{ J}$$

$$1 \text{ \AA} = 10^{-10} \text{ m}$$

$$hc = 12400 \text{ eV \AA}$$

$$\hbar c = 1973 \text{ eV \AA}$$

**B. Selected formulae**

Differential cross section for Rutherford scattering:

$$\frac{d\sigma}{d\Omega} = \frac{(Z'Z)^2 q^4}{256\pi^2 \epsilon_0^2 E^2} \frac{1}{\sin^4(\theta/2)}$$

Compton shift

$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

Radius of allowed orbits in Bohr model of hydrogenic atom:

$$r_n = \frac{n^2}{Z} a_0,$$

where  $a_0 = \frac{4\pi\epsilon_0 \hbar^2}{q^2 m} = 0.529 \text{ \AA}$  is the Bohr radius.

Ground-state energy of hydrogen:

$$E_1 = -13.6 \text{ eV}$$

Solid angle of thin ring of width dθ at angle θ:

$$d\Omega = 2\pi \sin \theta \, d\theta$$

Fourier transform:  $\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ikx} dx$        $\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{ikx} dk$

**1. Short problems** (30 points)

Please compute numerical answers to two significant digits.

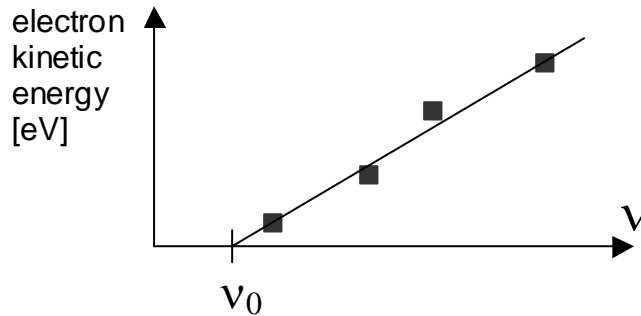
a) **Rutherford scattering** (10 points).

(i) (5 points) In the Rutherford scattering experiment, a beam of  $\alpha$ -particles impinges upon a thin gold foil. Find the ratio of the rate of particles that scatter into a thin ring of thickness  $d\theta$  at  $\theta = 3\pi/4$  to those that scatter into a thin ring of thickness  $d\theta$  at  $\theta = \pi/4$ .

ii) (5 points) Explain in one or two sentences what a differential cross section represents.

b) **Photoelectric effect** (12 points)

A measurement of the photoelectric effect for a copper metal plate yields the following result:



The best linear fit yields a slope of  $4.31 \times 10^{-15}$  eV/Hz and a cut-off frequency value  $\nu_0 = 1.12 \times 10^{15}$  Hz.

- i) (6 points) Explain in one or two sentences why no photoelectrons are observed for  $\nu < \nu_0$ .
- ii) (6 points) Determine Planck's constant and the work function of copper from this measurement (both in SI units).

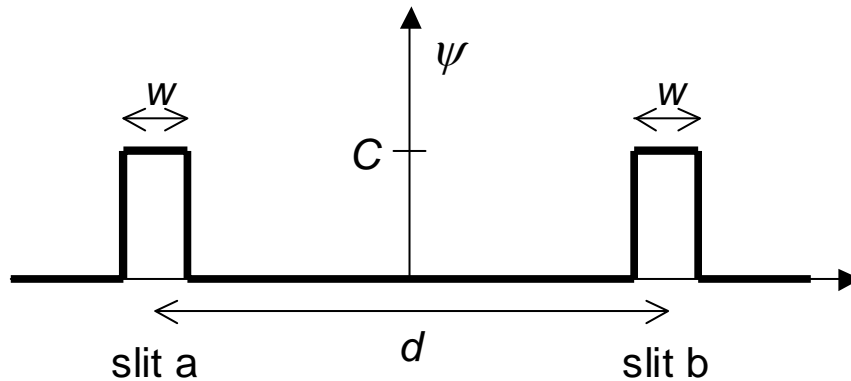
c) **Compton effect with protons?** (8 points)

We would like to determine if it is possible to observe the equivalent of the Compton effect for a target proton rather than a target electron. Calculate the maximum fractional frequency shift for an incident photon of wavelength  $\lambda = 1 \text{ \AA}$  scattering off a proton initially at rest. You may use a result known for a target electron in combination with a physical argument.

2. **Double slit.** (30 points)

The wavefunction of a particle in a double slit experiment with slit spacing  $d$  and slit width  $w < d$  ( $w, d$  both positive quantities) in the plane of the slits is described by

$$\psi(x) = \begin{cases} C & \text{for } -\frac{d}{2} - \frac{w}{2} \leq x \leq -\frac{d}{2} + \frac{w}{2} & \text{(slit a)} \\ C & \text{for } \frac{d}{2} - \frac{w}{2} \leq x \leq \frac{d}{2} + \frac{w}{2} & \text{(slit b)} \\ 0 & \text{otherwise} \end{cases}$$



- (5 points) Determine the normalization constant  $C$ .
- (5 points) Determine  $\langle x \rangle$  and  $\langle x^2 \rangle$  in the limit  $w \ll d$ , i.e. ignore any terms of order  $w/d$  and higher in the end result.
- (10 points) Calculate the probability density to measure a particle momentum  $p \in [p, p+dp]$ . Do not assume  $w \ll d$ .
- (10 points) Explain qualitatively how the interference pattern on a far-away screen changes if the wavefunction is modified as

$$\psi(x) = \begin{cases} C & \text{for } -\frac{d}{2} - \frac{w}{2} \leq x \leq -\frac{d}{2} + \frac{w}{2} & \text{(slit a)} \\ C e^{i\varphi} & \text{for } \frac{d}{2} - \frac{w}{2} \leq x \leq \frac{d}{2} + \frac{w}{2} & \text{(slit b)} \\ 0 & \text{otherwise} \end{cases}$$

with some fixed (real) phase  $\varphi$ . No calculation, or at most a simple calculation for the double slit, as shown in class, is necessary. You may assume that  $w$  is small compared to the deBroglie wavelength  $\lambda_{dB}$  of the particle, while  $d \gg w, \lambda_{dB}$ .

**3. Scattering and Heisenberg uncertainty** (20 points)

In Rutherford scattering, the structure of the atom is probed by  $\alpha$ -particle scattering, revealing the existence of the nucleus. Similarly, the structure of the nucleus itself can be probed, e.g., by electron scattering.

- a) (10 points) Assuming that in electron scattering off a target proton you need to resolve a distance scale below  $R=10^{-15}$  m, determine a condition on the electron momentum, and justify it in one or two sentences. Decide if this corresponds to a relativistic or a non-relativistic situation.
- b) (5 points) Calculate the electron energy necessary to resolve the structure of the proton in electron scattering. Assume that the proton target is at rest.
- c) (5 points) How much more or less particle energy would you need to resolve the same distance scale in proton-proton scattering ( $m_p=1800m_e$ )? Based on this, would you rather build an electron-proton, or a proton-proton collider?

**4. Predictive power of Heisenberg uncertainty** (20 points)

The Heisenberg uncertainty is often viewed as a limitation to measurements, but in combination with energy minimization it also has significant power in predicting the magnitude of quantum mechanical effects. Assume that you have a particle of mass  $m$  in a harmonic potential

$$V(x) = \frac{1}{2}m\omega^2x^2.$$

- a) (8 points) Argue in one or two sentences why a particle localized in a very narrow region around  $x=0$  would have a large total energy.
- b) (12 points) By minimizing the total energy subject to the Heisenberg uncertainty relation between position spread  $\Delta x$  and momentum spread  $\Delta p$  of the particle, calculate the size  $\Delta x$  of the ground state in the harmonic potential.