

12.003 Physics of Atmospheres and Oceans

Problem Set 2: Hydrostatic balance and lapse rates

Due date: Friday 28th September, 2007

Read Chapter 3 and Section 4.2 to 4.4 of Chapter 4 of our notes

1. Consider a horizontally uniform atmosphere in hydrostatic balance. The atmosphere is isothermal, with temperature of -10°C . Surface pressure is 1000hPa.
 - (a) Consider the level that divides the atmosphere into two equal parts by mass (*i.e.*, one-half of the atmospheric mass is above this level). What is the altitude, pressure, density and potential temperature at this level? If an air parcel were brought down to the surface, adiabatically, from this level, what would its temperature be at the surface?
 - (b) Repeat the calculation of part (a) for the level below which lies 90% of the atmospheric mass.
2. Derive an expression for the hydrostatic atmospheric pressure at height z above the surface in terms of the surface pressure p_s and the surface temperature T_s for: (i) an isothermal atmosphere at temperature T_s and (ii) an atmosphere with constant lapse rate of temperature $\Gamma = -\frac{dT}{dz}$. Express your results in terms of the dry adiabatic lapse rate $\Gamma_d = \frac{g}{c_p}$.
3. Somewhere (in a galaxy far, far away) there is a planet whose atmosphere is just like that of the Earth in all respects but one—it contains no moisture. The planet's troposphere is maintained by convection to be neutrally stable to vertical displacements. Its stratosphere is in radiative equilibrium, at a uniform temperature -80°C , and temperature is continuous across the tropopause. If the surface pressure is 1000hPa, and equatorial surface temperature is 32°C , what is the pressure at the equatorial tropopause?
4. For a perfect gas undergoing changes dT in temperature and dV in specific volume, the change ds in specific entropy, s , is given by

$$T ds = c_v dT + p dV .$$

- (a) Hence, for unsaturated air, show that potential temperature θ

$$\theta = T \left(\frac{p_0}{p} \right)^\kappa ,$$

(notation defined in notes) is a measure of specific entropy; specifically, that

$$s = c_p \ln \theta + \text{constant} .$$

where c_v and c_p are specific heats at constant volume and constant pressure, respectively.

- (b) Show that if the environmental lapse rate is dry adiabatic, then it has constant potential temperature.
5. Assume the atmosphere is isothermal with temperature 280K. Determine the potential temperature at altitudes of 5km, 10km, and 20km above the surface. If an air parcel were moved adiabatically from 10km to 5km, what would its temperature be on arrival?