14.126 Game Theory

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Lecture 9:

The Rubinstein-Stahl Model
(perfect info. noncooperative bargaining, cont'd)
Cooperative Nash Bargaining

Bargaining Theory

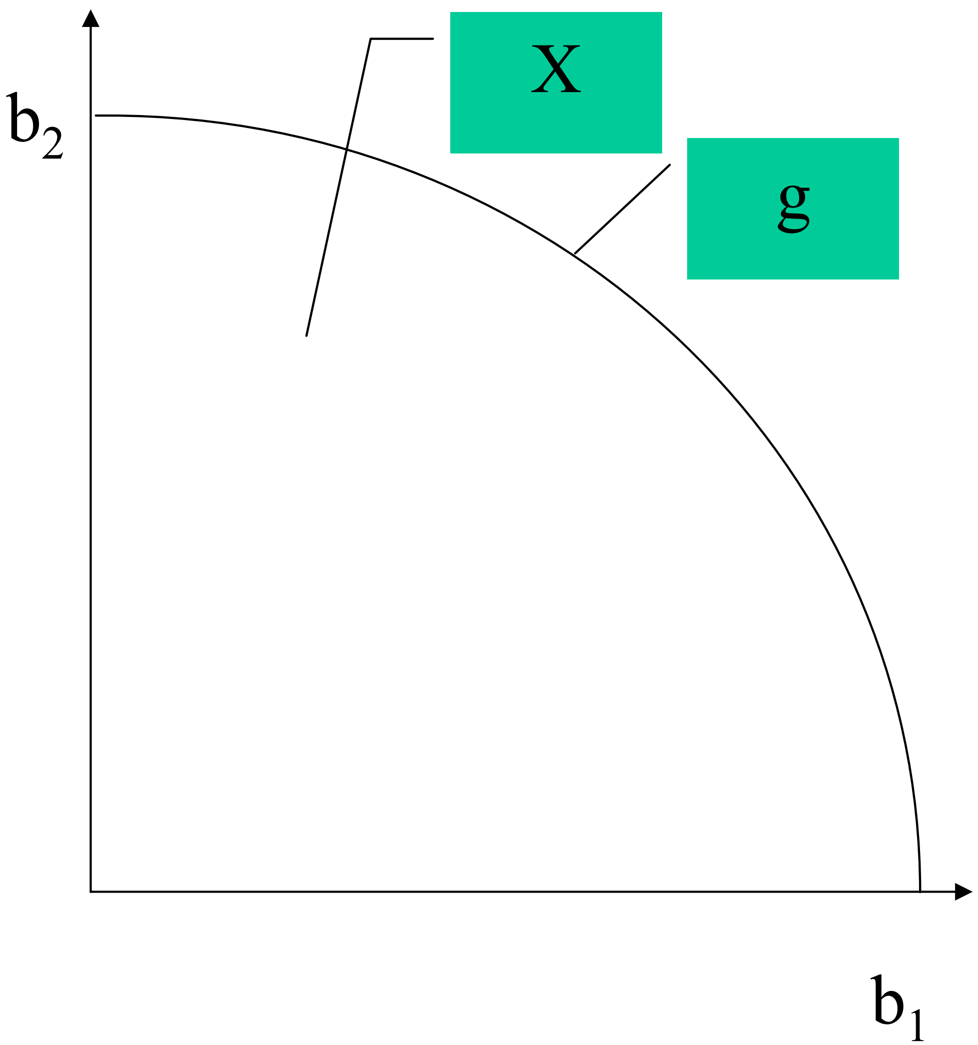
Slides: Courtesy of Paul Milgrom and Muhamet Yildiz

Bargaining Theory

- Cooperative (Axiomatic)
  - Edgeworth
  - Nash Bargaining
  - Variations of Nash
  - Shapley Value

- Non-cooperative
  - **Rubinstein-Stahl (*)** (complete info)
  - Asymmetric info
    - Rubinstein, Admati-Perry, Crampton, **Gul, Sonenchein, Wilson; Abreu and Gul**
  - Non-common priors
    - Posner, Bazerman, **Yildiz (*)**

Rubinstein-Stahl Model



- N = {1,2}
- X = feasible expected-utility pairs (x,y $\in$X)
- $U_i(x,t) = \delta_i^t x_i$
- D = (0,0) $\in$X disagreement payoffs
- g is concave, continuous, and strictly decreasing

Timeline

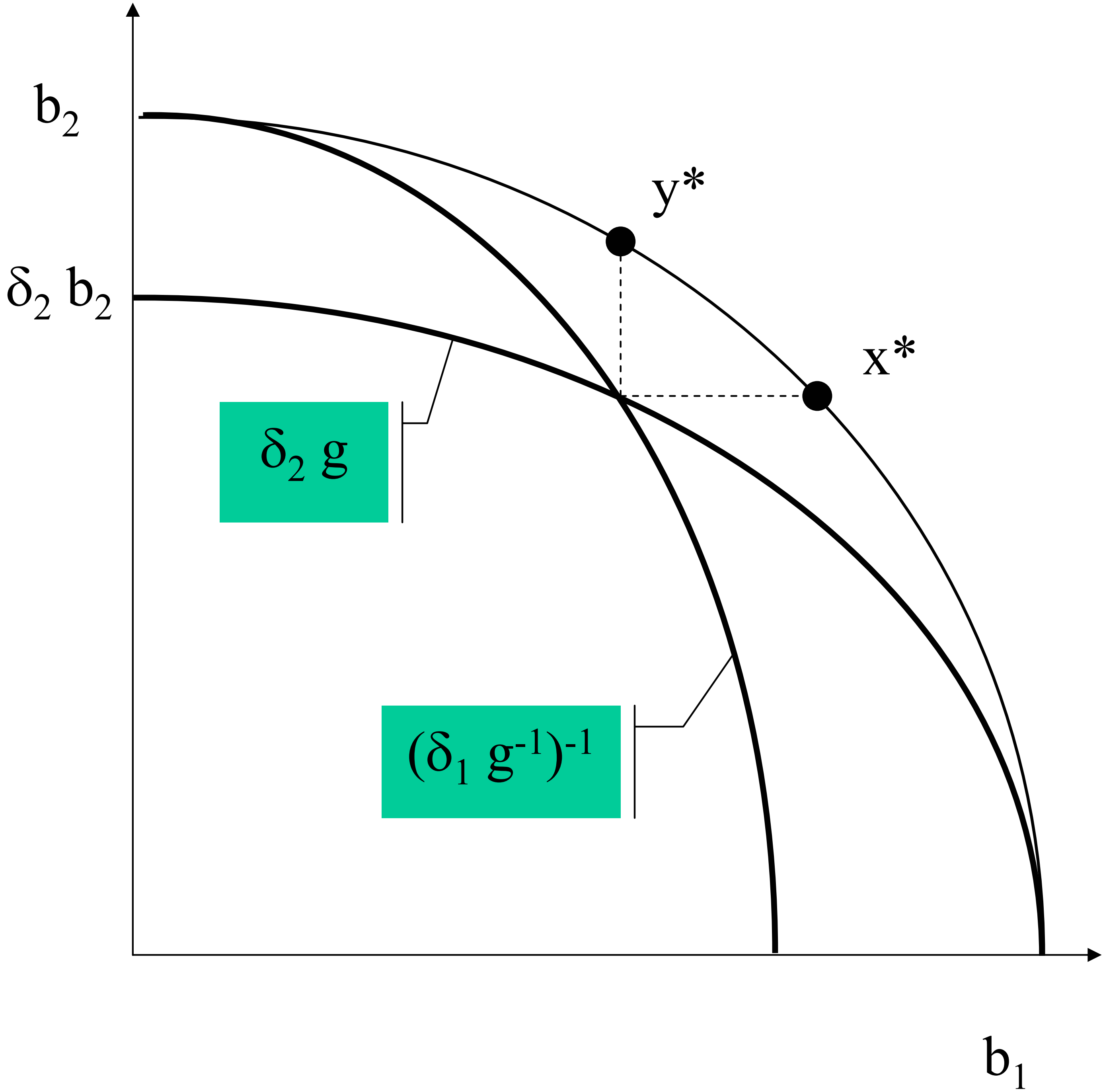
$T = \{0,1,\ldots, t,\ldots\}$

At each t, if t is even,

- Player 1 offers some x
- Player 2 Accepts or Rejects the offer
- If the offer is Accepted, the game ends yielding x
- Otherwise, we proceed to t + 1

if t is odd,

- Player 2 offers some y
- Player 2 Accepts or Rejects the offer
- If the offer is Accepted, the game ends yielding y
- Otherwise, we proceed to t + 1



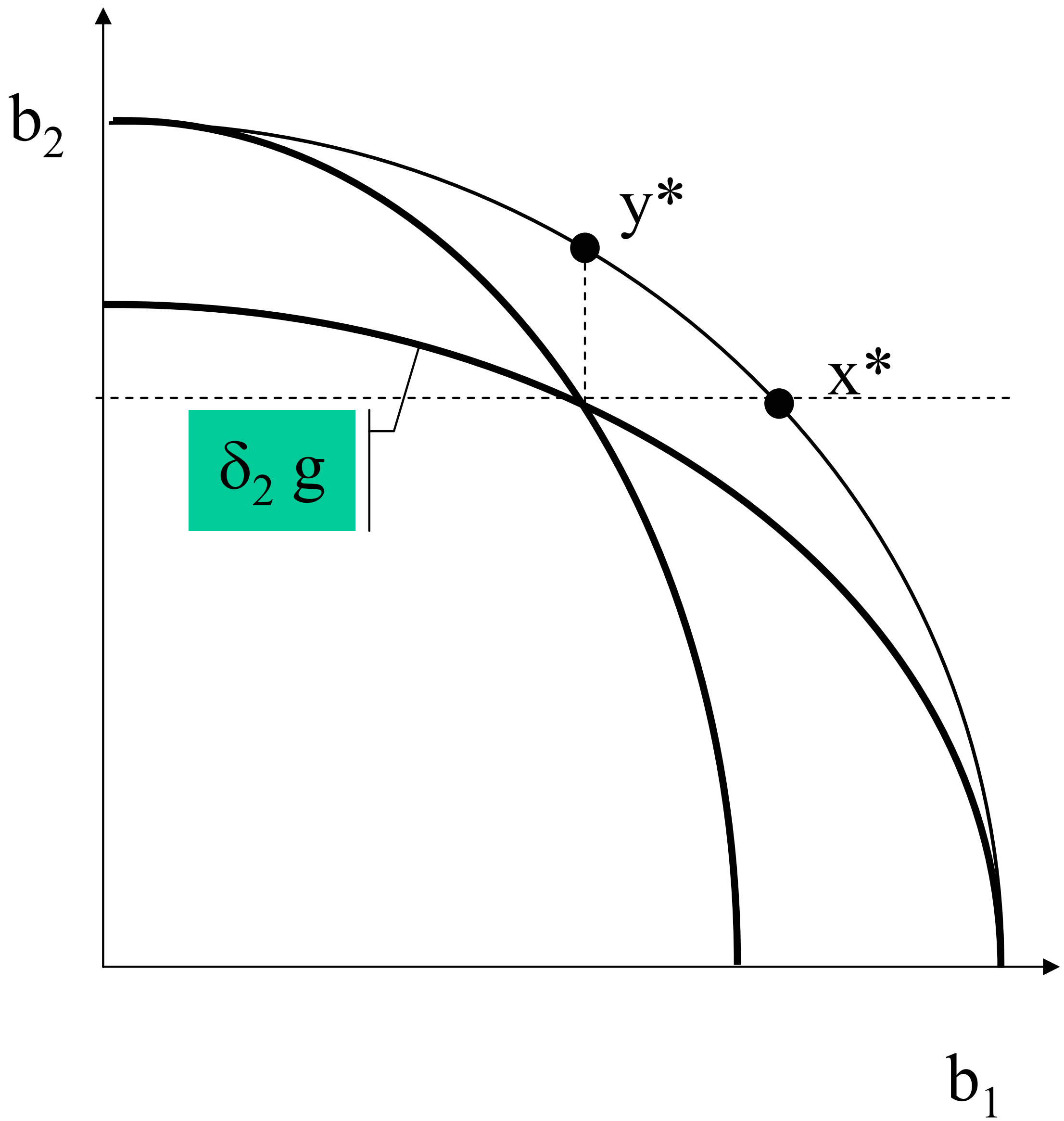
b_2
$\delta_2 b_2$
y*
x*
$\delta_2 g$
$(\delta_1 g^{-1})^{-1}$
b_1

Theorem [OR 122.1]

The following is the unique subgame-perfect equilibrium:

- player 1 always offers x*;
- player 2 accepts an offer x iff $x_2 \geq x^*_2$;
- player 2 always offers y*;
- player 1 accepts an offer y iff $y_1 \geq y^*_1$;

Proof (it is a SPE)



Use single deviation principle:

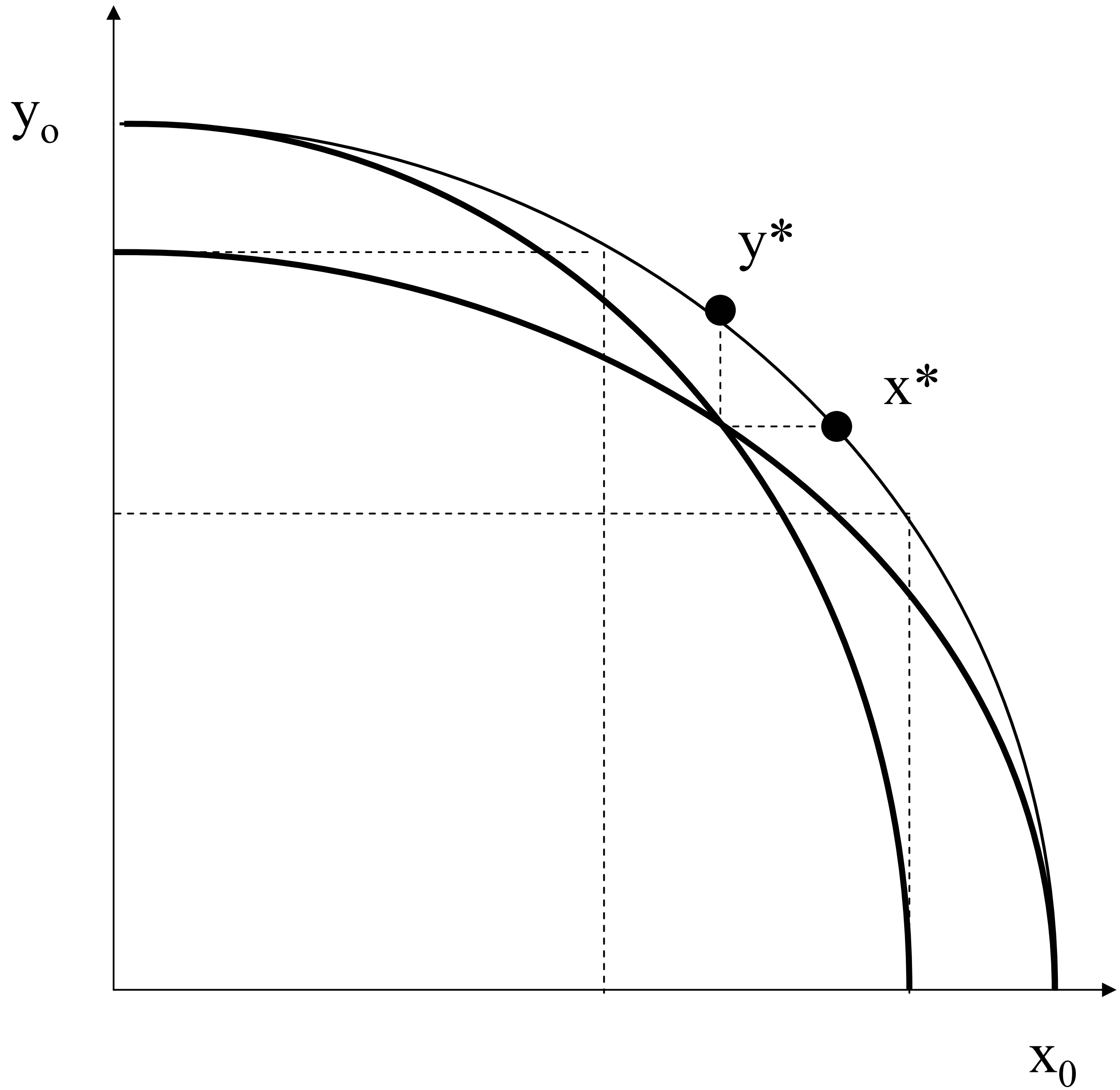
1. If player 2 rejects an offer x at t, she will get y_2* at t+1. Hence, Accept iff $x_2 \geq \delta_2 y^*_2 = x^*_2$ is optimal at t.
2. At t, it is optimal for 1 to offer

$x^* = \text{argmax } \{x_1 | x_2 \geq x^*_2\}$.

"Extensive-form rationalizability" [FT 4.6]

Definition: In a multistage game with observable actions, action a_i^t is conditionally dominated at stage t given history h^{t-1} iff, in the subgame starting at h^{t-1}, every strategy for player i that assigns positive probability to a_i^t is strictly dominated.

Theorem: In any perfect-information game, every subgame-perfect equilibrium survives iterated elimination of conditionally dominated strategies.


y*
x*
y_o
x_0

Cooperative Bargaining

Nash Bargaining Problem

- $N = \{1,2\}$ – the agents
- $S \subset R^N$ -- the set of feasible expected-utility pairs
- $d = (d_1,d_2) \in S$ – the disagreement payoffs
- A *bargaining problem* is any (S,d) where
  - S is compact and convex, and
  - $\exists x \in S$ s.t. $x_1 > d_1$ and $x_2 > d_2$.
- B is the set of all bargaining problems.
- A *bargaining solution* is any function $f : B \to R^N$ s.t. $f(S,d) \in S$ for each (S,d).

Nash Axioms

1. **Expected-utility Axiom [EU]** (invariance under affine transformations): $\forall(S,d), \forall(S',d'), a_\mathrm{i} > 0$

$$\left.\begin{array}{l} S' \quad \{s' \mid s'_i \quad a_i s_i + b_i \ \forall i \in N\} \\ d'_i \quad a_i d_i + b_i \ \forall i \in N \end{array}\right\} \Rightarrow f_i(S',d') \quad a_i f_i(S,d) + b_i \ \forall i \in N$$

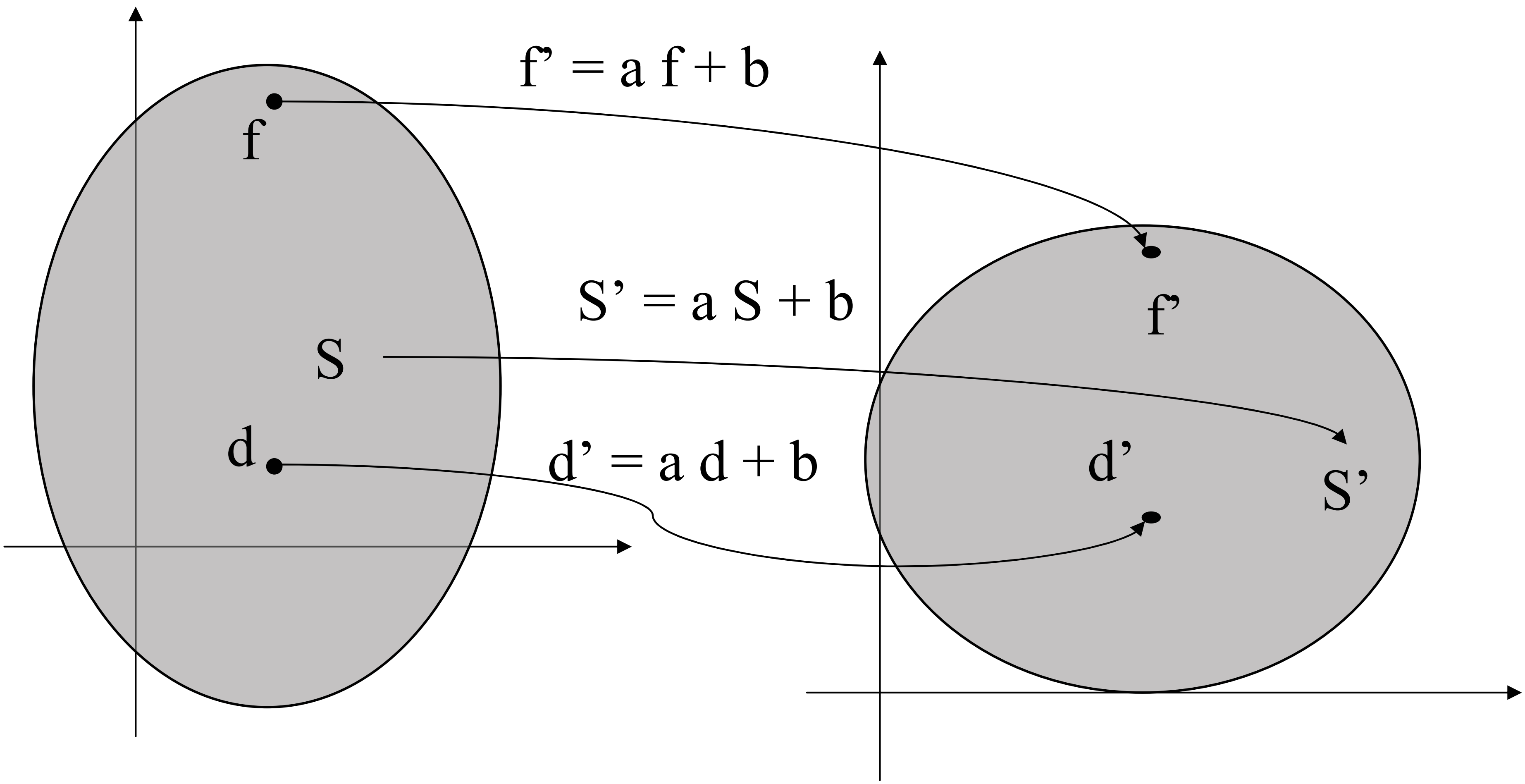
2. **Symmetry [Sy]:** Let (S,d) be symmetric: $d_1 = d_2$ and $[(x_1,x_2) \in S$ iff $(x_2,x_1) \in S]$. Then,

$$f_1(S,d) = f_2(S,d).$$

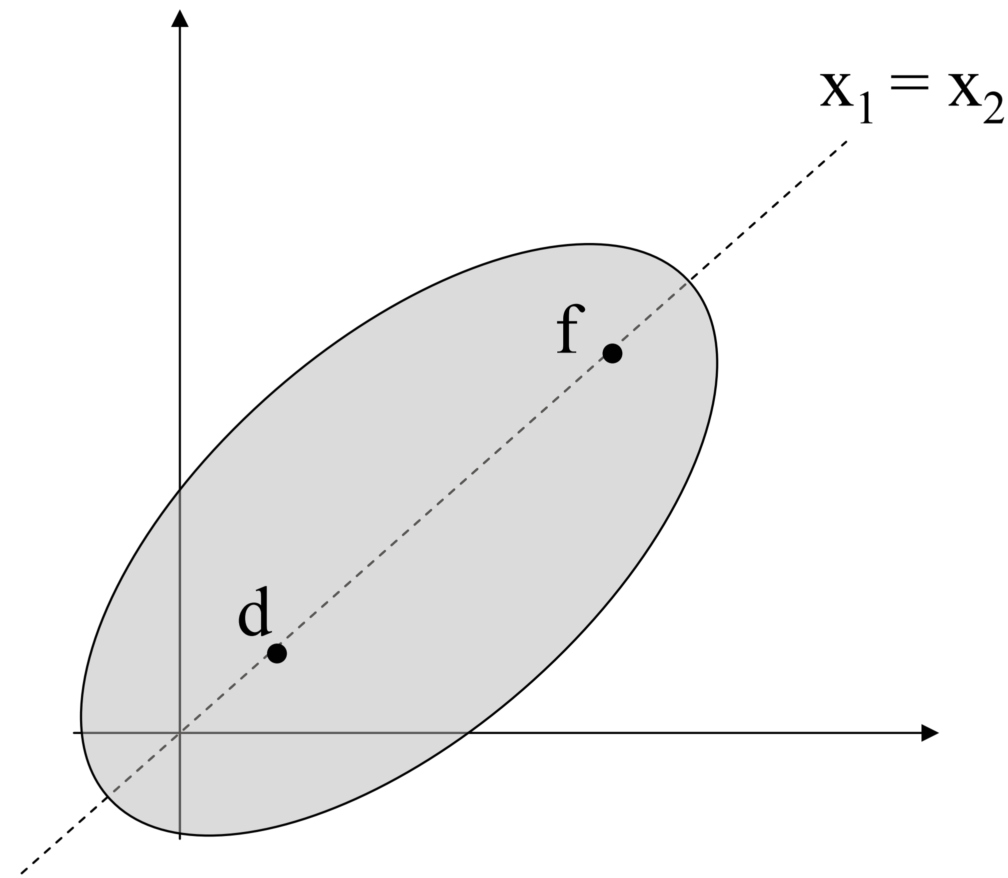
3. **Independence of Irrelevant alternatives [IIA]**: if $T \subset S$ and $f(S,d) \in T$, then $f(T,d) = f(S,d)$.

4. **Pareto – Optimality [PO]:** if $x,y \in S$ and $y > x$, then $f(S,d) \neq x$.

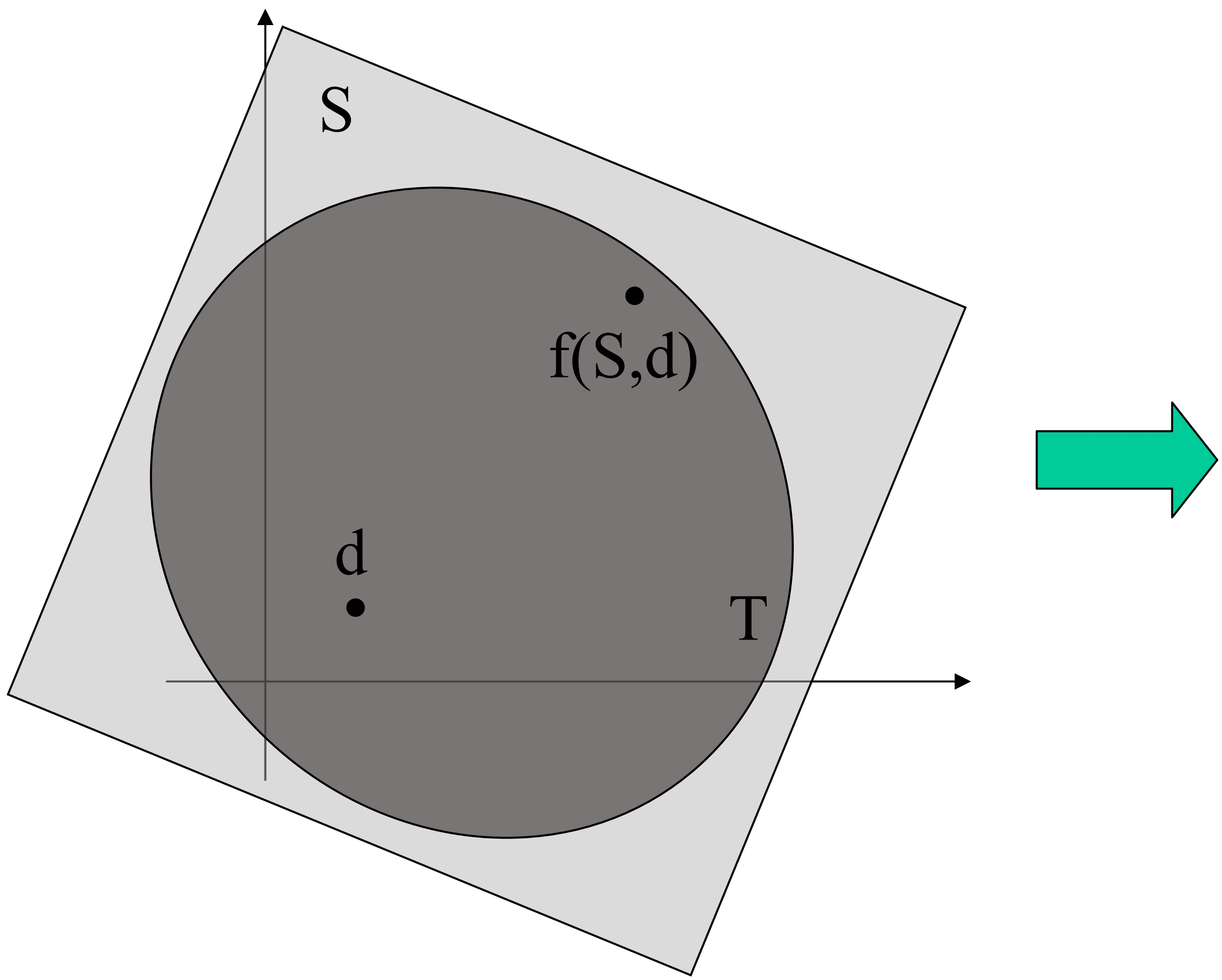
Expected-utility Axiom



Symmetry



Independence of irrelevant alternatives

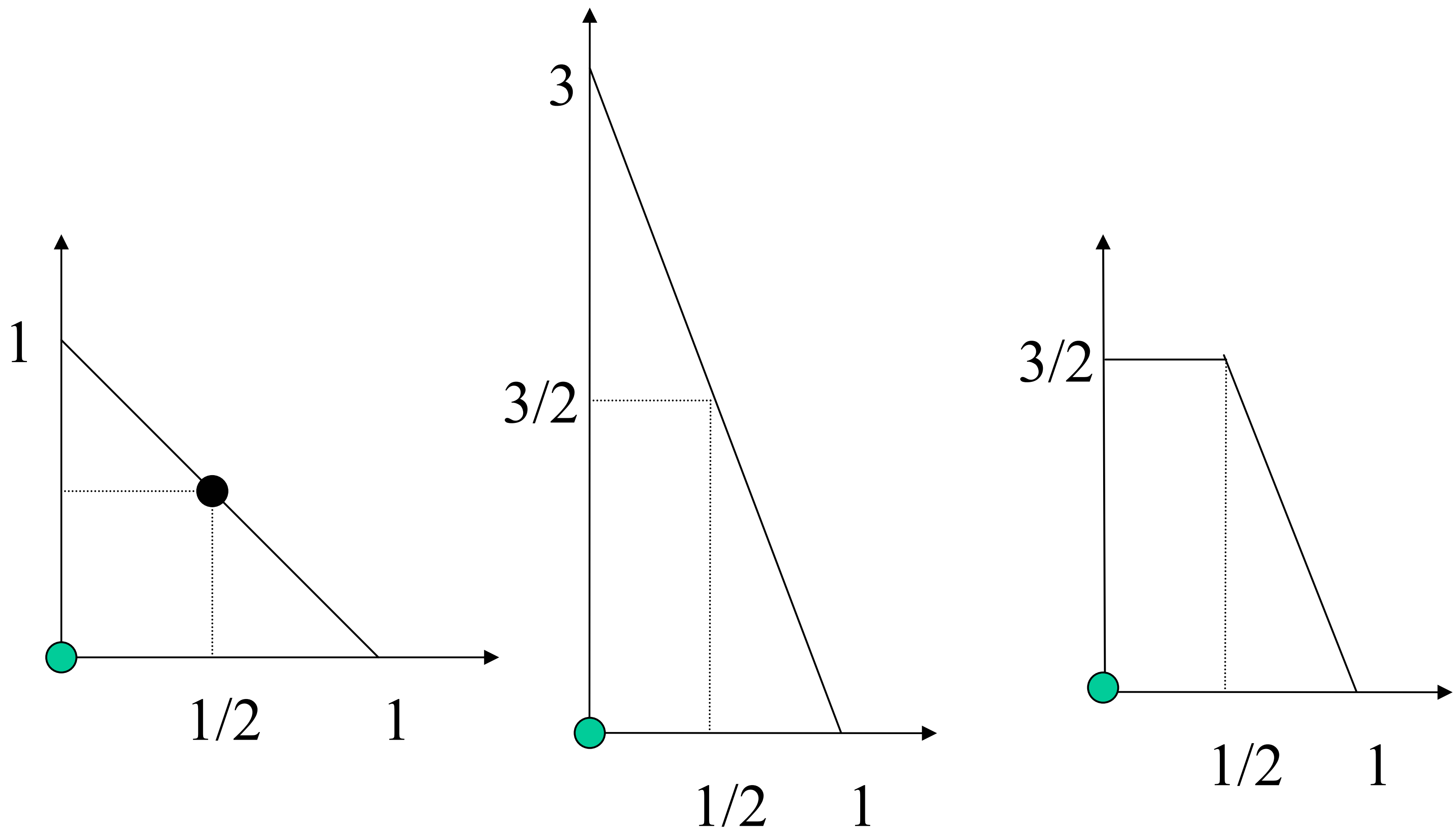


f(T,d) = f(S,d)

Nash Bargaining Solution

$$f^*(S,d) \quad \underset{\substack{s\equiv(s_1,s_2)\in S \\ s>d}}{\arg\max}\,(s_1 - d_1)(s_2 - d_2).$$

Examples



$$f^*(S,d) \quad \underset{\substack{s\equiv(s_1,s_2)\in S \\ s>d}}{\arg\max}\,(s_1-d_1)(s_2-d_2).$$

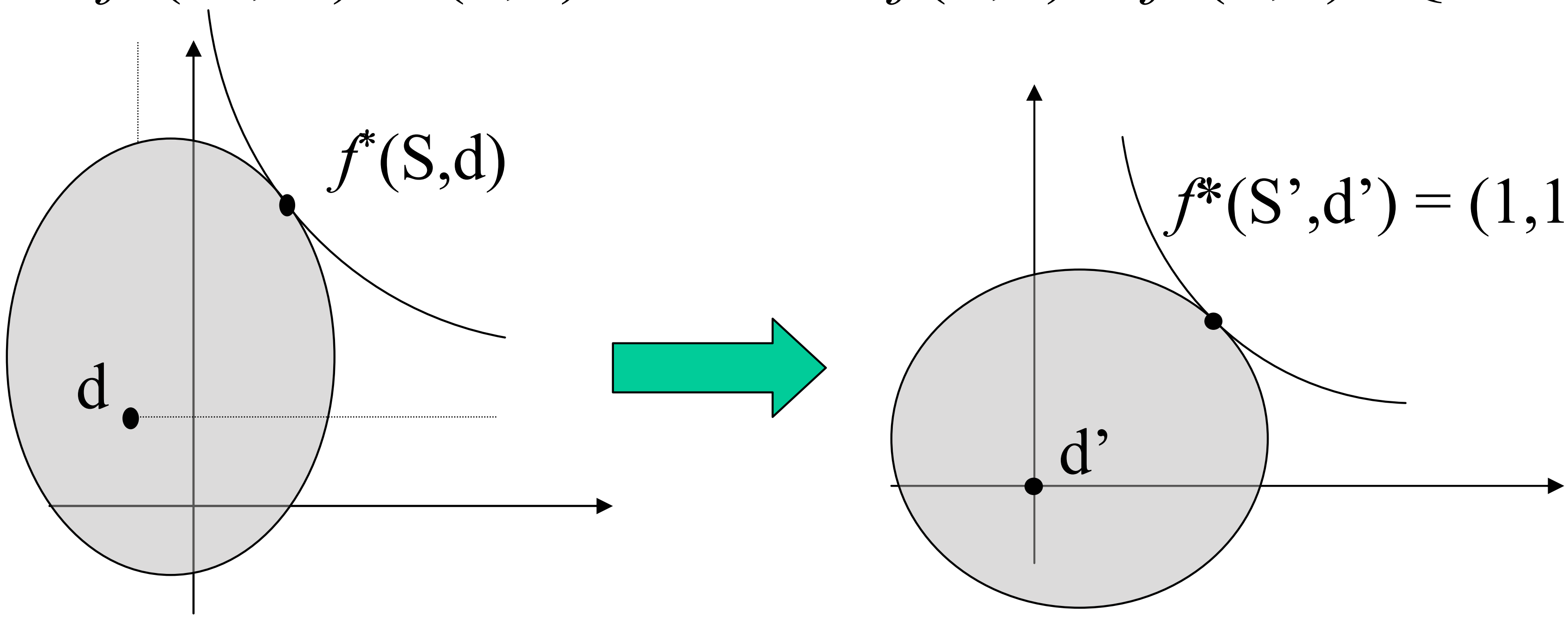
Nash's Theorem

Theorem: A bargaining solution f satisfies the Nash Axioms (EU,Sy,IIA,PO) if and only if

$$f = f^*.$$

Proof of Nash's Theorem

1. Check: f^* satisfies the Nash axioms. (easy)
2. Take any (S,d). Transform it to (S',d') so that d' = 0, and f^*(S',d') = (1,1). Under [Sy,IIA,PO], f(S',d') = f^*(S',d') = (1,1). &EU $\Rightarrow$ f(S,d) = f^*(S,d). QED



- S^* is symmetric. (how)
- &Sy&PO $\Rightarrow f(S^*,d') = (1,1) \in S'$
- &IIA $\Rightarrow f(S',d') = (1,1) = f^*(S',d')$
- &EU $\Rightarrow f(S,d) = f^*(S',d')$