14.126 Game Theory — Homework 4

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Due on 12/3 Friday [can be extended on request]

1. Consider a finite game (N, A, u). Show that the rationalizable outcomes of this complete-information game, denoted by R, can be taken to be equilibrium outcomes of a game with private information. That is,

$$s^*(\Omega) = R$$

for some family $(\Omega, \mathcal{I}_i, p_i)_{i \in N}$ of probability spaces and equilibrium s^*, where $\mathcal{I}_i$ is a partition of Ω, and $s^* : \Omega \to A$ is such that, for each i, s_i^* is $\mathcal{I}_i$-measurable, and there is no $\mathcal{I}_i$-measurable function s_i that gives expected payoff higher than that of s_i^* given that the others play according to s_{-i}^*.

2. This question illustrates that the epistemic model developed in the class can be used to analyze the strategic behavior of "irrational" players. It is based on Yildiz, "Wishful thinking in strategic environments." Consider a two-player, finite action game (N, A, u). Consider the "event"

$$\text{Player 1 is a wishful thinker and Player 2 is rational.} \qquad \text{(W)}$$

(Given any model $\left(\Omega, \{\mathcal{I}_i\}_{i \in N}, \{p_i\}_{i \in N}, s\right)$, a player i is said to be a wishful thinker at a state ω if

$$E_{i,\omega}[u \circ s] = \max_{s \in A_i \times s_{-i}(I_i(\omega))} u_i(s),$$

where $E_{i,\omega}$ denotes the expectation w.r.t. $p_i(\cdot | I_i(\omega))$.)

   (a) Characterize the strategy profiles that are consistent with W.
   (b) Characterize the strategies that are consistent with mutual knowledge of W.
   (c) Describe an iterated elimination process that yields the strategy profiles that are consistent with kth-order mutual knowledge of W for each k.

(d) **Bonus:** Assuming that u is generic, relate the strategy profiles that are consistent with common knowledge of W to Nash equilibrium strategies.

3. This question asks you to apply global games logic to analyze collective action. Consider a country occupied by a foreign force. Assuming that individuals are small compared to the total population, model the population as unit interval, $[0,1]$. Each individual $i \in [0,1]$ is to decide whether to revolt or not. Observing the size of people who revolt, the occupier decides whether to suppress the insurgency or end the occupation. Payoffs depend on what would happen when the occupation ends. A wide range of outcomes are possible, including a consolidated strong democracy as in Japan after the World War II, or a civil war that exports terrorism as in Afghanistan after the Russian invasion. The state of the country after occupation is summarized by θ, which can be any real number. If occupation ends, the foreign force gets $b(\theta)$ where b is strictly increasing, $\lim_{\theta \to -\infty} b(\theta) = -\infty$, and $\lim_{\theta \to \infty} b(\theta) = \infty$. For the foreign force, the benefit of occupation is v, and the cost of suppressing insurgency is αc, where α is the size of population who revolt and $c > 0$. The payoff for an individual is θ if the occupation ends, and 0 if occupation continues. Each individual also has a direct benefit/cost of revolting. If an individual revolts, then he gets $B > 0$ if occupation ends and $-C \ll 0$ if the insurgency is suppressed.

   (a) Assume that θ is common knowledge and find all equilibria.

   (b) Assume that occupying force know θ but each individual only gets a noisy signal $s_i = \theta + \varepsilon_i$ where each ε_i is distributed with $N(0, \sigma^2)$, θ is distributed with $N(0,1)$, and $\{\theta\} \cup \{\varepsilon_i : i \in N\}$ are stochastically independent. Compute rationalizable strategies. (You can assume that σ^2 is very small if you want.)

   (c) Assume that the population consists of two tribes, $t_1 = [0, 1/2)$ and $t_2 = [1/2, 1]$. Each tribe has a member, called the chief, who decides for all other members, using his own signal. Under the informational assumptions of part (b), compute all Bayesian Nash equilibria.

   (d) **Bonus:** Assuming that occupier does not know θ but gets a noisy signal $s_o = \theta + \varepsilon_o$ with independent noise ε_o distributed with $N(0, \sigma^2)$, repeat part (b).