

14.126 Game Theory Problem Set 3

Due on 11/23/2004

1. Consider the following Hawk-Dove game, where doves also incur some small cost t when they meet each other:

| | | |
|---|----------------------------|--------------------|
| | H | D |
| H | $(v - w) / 2, (v - w) / 2$ | $v, 0$ |
| D | $0, v$ | $v/2 - t, v/2 - t$ |

- (a) Assuming that $w > v$ and $t > 0$ find all evolutionarily stable strategies.
 (b) Now consider a third type, B, who plays H if he is the first to come to a territory and plays D (without incurring the cost t) if he is the second:

| | | | |
|---|----------------------------|-------------------------|---------------------------|
| | H | D | B |
| H | $(v - w) / 2, (v - w) / 2$ | $v, 0$ | $3v/4 - w/4, (v - w) / 4$ |
| D | $0, v$ | $v/2 - t, v/2 - t$ | $v/4 - t/2, 3v/4 - t/2$ |
| B | $(v - w) / 4, 3v/4 - w/4$ | $3v/4 - t/2, v/4 - t/2$ | $v/2, v/2$ |

For which values of v , w , and t , B is an evolutionarily stable strategy?

2. Consider the following "ultimatum game:" Player 1 offers some $a \in A = \{0.01, 0.02, \dots, 1\}$, and Player 2 demands some $b \in A$. If $a \geq b$ (i.e., if Player 2 accepts the offer a), then the payoffs are $(1 - a, a)$; otherwise, the payoffs are $(0, 0)$.
- (a) Compute all Nash equilibria.
 (b) Now consider an evolutionary process in which the members of a population is matched in pairs to play this ultimatum game where each agent is equally likely to play the roles of Player 1 and Player 2. Assume that the growth of the strategies in this role-playing game follows the replicator dynamics. Find all asymptotically stable strategies.
3. Consider a knowledge operator K that satisfies the properties of (1) $KE \subseteq E$, (2) $E \subseteq F \Rightarrow KE \subseteq KF$, (3) $KKE = KE$, and (3) $K \sim KE = \sim KE$. Starting from K , define the associated knowledge function, information function, information "partition," and information filed. Show that the associated information partition is indeed a partition. Show also that they are indeed associated, i.e., if one starts from the derived information function and defines a knowledge operator K' in the same way we defined in the class, then $K' = K$.

4. Given any finite event E and two agents 1 and 2, show that there exists some integer \bar{n} such that for any chain $i_0 \neq i_1 \neq i_2 \neq \dots \neq i_{n-1} \neq i_n$ with $n \geq \bar{n}$, the event

$$F = K_{i_0} K_{i_1} K_{i_2} \dots K_{i_{n-1}} K_{i_n} E$$

is a public event between 1 and 2, i.e., $CK_{1,2}F = F$.