14.126 (2004) Problem Set #2 (Due: Oct 15, in Recitation)

Basics Concepts in Game Theory
& Repeated Games with Perfect Monitoring

1. Do the following exercises from the Osborne-Rubinstein book. In the exercises followed by a *, restrict attention to pure strategies.

   (a) (10pts. Rationalizability) 56.5.

   (b) (10pts. Pure Strategy NE) 19.1*. Assume that citizens are uniformly distributed, i.e. $f(x) = 1$.

   (c) (10pts. Mixed Strategy NE) 36.2. Assume that $v_1 = 3$, $v_2 = 2$, and $v_3 = 1$.

   (d) (10pts. Pure Strategy BNE) 28.1*.

   (e) (10pts. SPE) 103.2*.

   (f) (10pts. Forward Induction) 114.1*.

   (g) (10pts. Sequential Equilibrium) 226.1.

2. (Repeated Games with Perfect Monitoring) Consider the infinite repetition of a finite two-player normal form game G. Assume that there is no public randomization device and restrict attention to pure strategies.

   (a) (10pts) Suppose that $\bar{a}$ is a strictly enforceable action profile such that:

   $$(*) \quad u_i(a_i, \bar{a}_{-i}) - u_i(\bar{a}) < u_i(\bar{a}) - u_i(p_i, p_{-i}) \quad \forall i \in N, a_i \in A_i,$$

   where p_j denotes an action of j that min-maxes player $-j$. Construct a machine with two states that implements an SPE of $G^\delta(\infty)$ for δ close enough to 1, where $\bar{a}$ is played on the equilibrium path.

   (b) (10pts) Suppose now that $(*)$ need not hold. Modify the machine in the previous part by adding more states such that it implements an SPE of $G^\delta(\infty)$ for δ close to 1, where $\bar{a}$ is played on the equilibrium path .

   (c) (10pts) Show that the NEU condition is not necessary for the folk theorem if the NSM condition is not satisfied.

3. (15pts. Existence of Symmetric NE) A normal form game $G = (N, A, u)$ is **weakly symmetric** if (i) there is a set X such that every player i has the same set of actions $A_i = X$, and (ii) for any $i, j \in N$ and $\alpha, \beta \in \Delta(X)$, we have:

$$u_i(\sigma) = u_j(\sigma')$$

where $\sigma, \sigma' \in \Delta(X)^N$ are given by:

$$\sigma_k = \begin{cases} \alpha & \text{if } k = i \\ \beta & \text{if } k \neq i \end{cases} \qquad \text{and} \qquad \sigma'_k = \begin{cases} \alpha & \text{if } k = j \\ \beta & \text{if } k \neq j \end{cases}$$

A NE σ^* of G is called **symmetric** if $\sigma_1 = \sigma_2 = \ldots = \sigma_n$. Show that every weakly symmetric finite normal form game has a symmetric NE.

4. (30pts, Difficult. Weak Domination) Prove that in a finite normal form game, an action a_i^* is never a best reply to any (possibly correlated) completely mixed conjecture σ_{-i} of i, if and only if a_i^* is weakly dominated to a mixed strategy σ_i.