Lecture 5
Solution concepts

14.12 Game Theory

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Road Map

1. Dominant-strategy equilibrium
2. Rationalizability
3. Nash Equilibrium

Dominance

$s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$

Definition: A pure strategy s_i^* **strictly dominates** s_i if and only if

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i}.$$

A mixed strategy σ_i **strictly dominates** s_i iff

$$\sigma_i(s_{i1})u_i(s_{i1}, s_{-i}) + \cdots + \sigma_i(s_{ik})u_i(s_{ik}, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i}$$

A rational player never plays a strictly dominated strategy.

Prisoners’ Dilemma

1 \ 2	Cooperate	Defect
Cooperate	(5,5)	(0,6)
Defect	(6,0)	(1,1)

Weak Dominance

Definition: A pure strategy s_i* weakly **dominates** s_i if and only if

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall s_{-i}$$

and at least one of the inequalities is strict. A mixed strategy σ_i* **weakly dominates** s_i iff

$$\sigma_i(s_{i1})u_i(s_{i1}, s_{-i}) + \cdots + \sigma_i(s_{ik})u_i(s_{ik}, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall s_i$$

and at least one of the inequalities is strict.

Dominant-strategy equilibrium

Definition: A strategy s_i* is a **dominant strategy** iff s_i* **weakly dominates** every other strategy s_i.

Definition: A strategy profile s* = (s_1*,…, s_n*) is a dominant-strategy equilibrium iff s_i* is a dominant strategy for each player i.

Prisoners' Dilemma

1 \ 2	Cooperate	Defect
Cooperate	(5,5)	(0,6)
Defect	(6,0)	(1,1)

Second-price auction

- $N = \{1,2\}$ buyers;
- The value of the house for buyer i is v_i;
- Each buyer i simultaneously bids b_i;
- i^* with $b_{i^*} = \max b_i$ gets the house and pays the second highest bid

$$p = \max_{j \neq i} b_j.$$

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2^{nd} price Auction

- Strtegies:

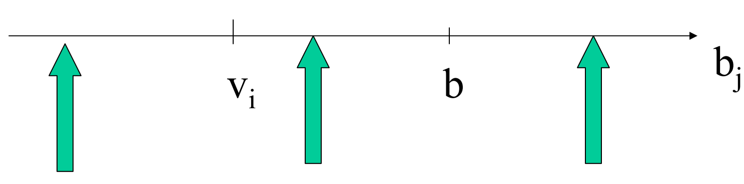
$$b_i \in [0,\infty)$$

- Payoffs:

$$u_i\,(b_i,b_j) = v_i - b_j \qquad \text{if } b_i > b_j$$
$$= (v_i - b_j)/2 \quad \text{if } b_i = b_j$$
$$= 0 \text{ otherwise.}$$

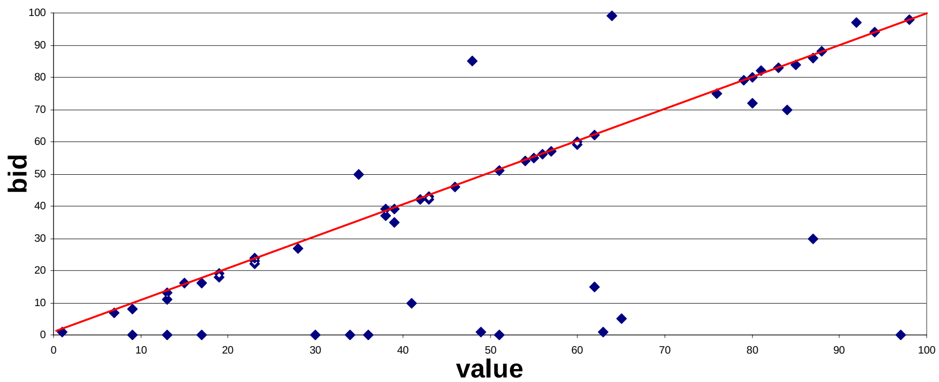
$b_i = v_i$ is a dominant strategy

$b_i = v_i$ dominates any $b > v_i$:



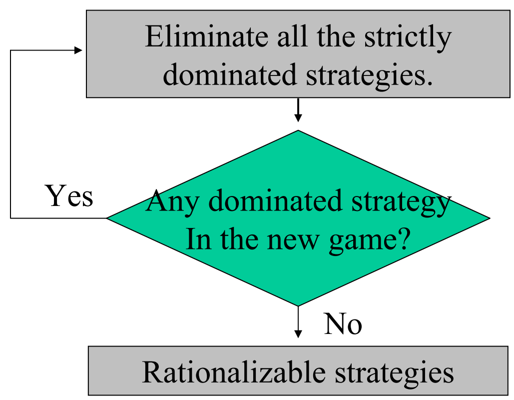
$u_i\,(v_i,b_j) = v_i\text{-}b_j$	$u_i\,(v_i,b_j) = 0$	$u_i\,(v_i,b_j) = 0$
$u_i\,(b,b_j) = v_i\text{-}b_j$	$u_i\,(b,b_j) = v_i\text{-}b_j < 0$	$u_i\,(b,b_j) = 0$

Bid Function in 2nd Price Auction



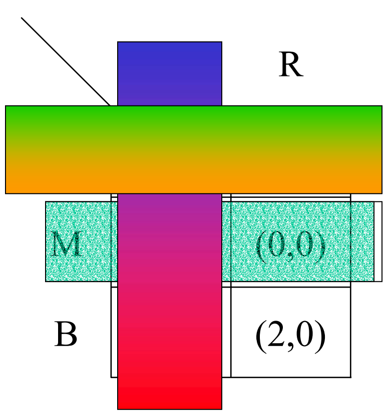
Rationalizability

Rationalizability



The play is rationalizable, provided that …

Assume



Player 1 is rational

Player 2 is rational

Player 2 is rational and
Knows that Player 1 is rational

Player 1 is rational,
knows that 2 is rational
knows that 2 knows that
1 is rational

Assume

1 \ 2	L	R
T	(3,0)	(0,3)
B	(0,3)	(3,0)

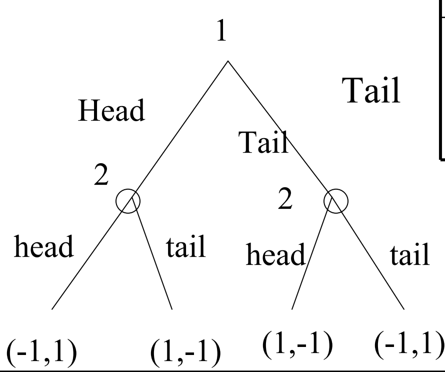
P1 is rational

P2 is rational and
knows that P1 is rational

P1 is rational and
knows all these

Matching pennies with perfect information

1 \ 2	HH	HT	TH	TT
Head	(-1,1)	(-1,1)	(1,-1)	(1,-1)
Tail	(1,-1)	(-1,1)	(1,-1)	(-1,1)



Nash Equilibrium

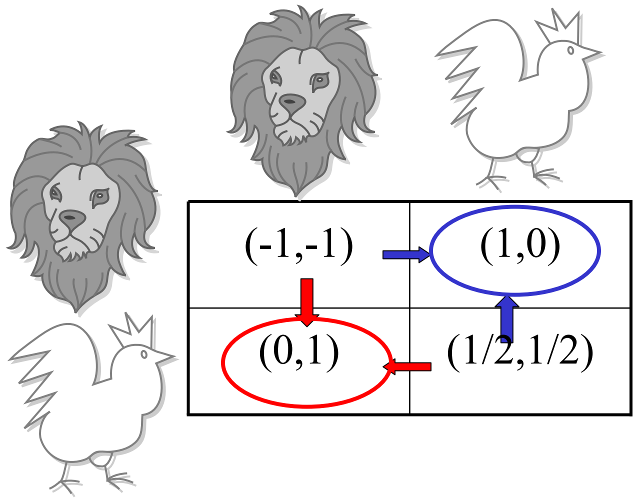
Nash Equilibrium

Definition: A strategy-profile $s^* = (s_1^*, \ldots, s_n^*)$ is a **Nash Equilibrium** iff, for each player i, and for each strategy s_i, we have

$$u_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*)$$
$$\geq u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*),$$

i.e., no player has any incentive to deviate if he knows what the others play.

Chicken



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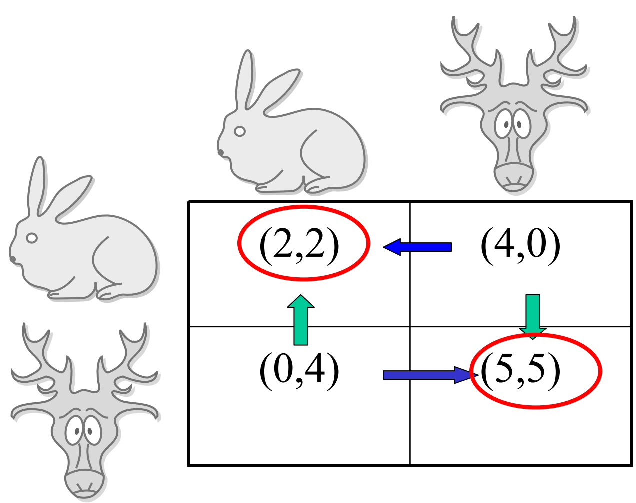
Stag Hunt



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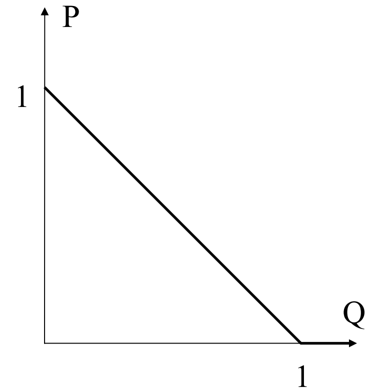
Cournot Oligopoly

- $N = \{1,2,\ldots,n\}$ firms;
- Simultaneously, each firm i produces q_i units of a good at marginal cost c,
- and sells the good at price

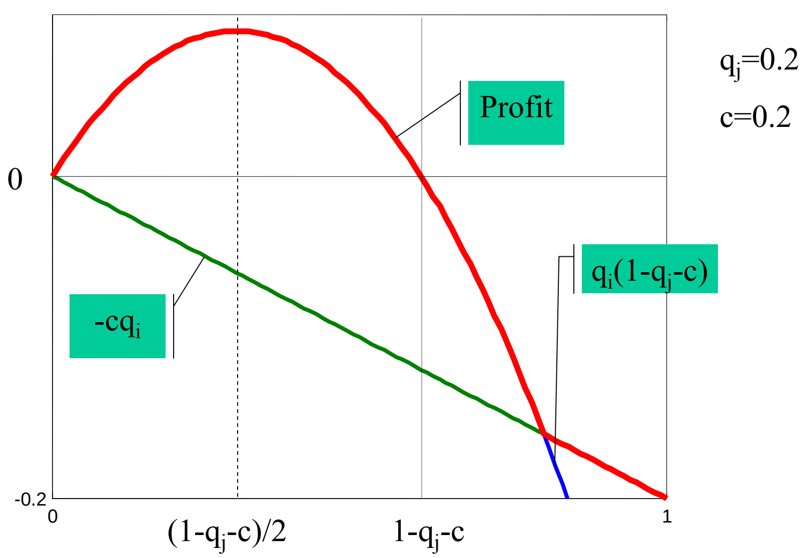
$$P = \max\{0, 1-Q\}$$

where $Q = q_1+\ldots+q_n$.
- Game $= (S_1,\ldots,S_n; \pi_1,\ldots,\pi_n)$ where $S_i = [0,\infty)$,



$$\pi_i(q_1,\ldots,q_n) = \begin{cases} q_i[1-(q_1+\ldots+q_n)-c] & \text{if } q_1+\ldots+qn < 1, \\ -q_i c & \text{otherwise.} \end{cases}$$

Cournot Duopoly -- profit



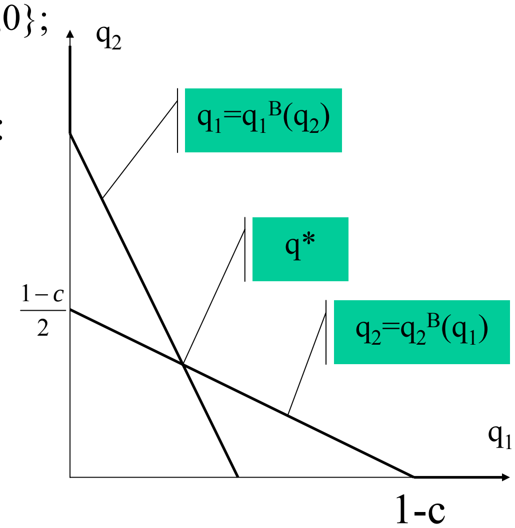
C-D – best responses

$q_i^B(q_j) = \max\{(1-q_j-c)/2, 0\}$;

- Nash Equilibrium q*:

  $q_1^* = (1-q_2^*-c)/2$;

  $q_2^* = (1-q_1^*-c)/2$;
- $q_1^* = q_2^* = (1-c)/3$



Cournot Oligopoly --Equilibrium

- $q > 1-c$ is strictly dominated, so $q \le 1-c$.
- $\pi_i(q_1,\ldots,q_n) = q_i[1-(q_1+\ldots+q_n)-c]$ for each i.
- FOC:

$$\left.\frac{\partial \pi_i(q_1,\ldots,q_n)}{\partial q_i}\right|_{q=q^*} = \left.\frac{\partial[q_i(1-q_1-\cdots-q_n-c)]}{\partial q_i}\right|_{q=q^*}$$

$$= (1-q_1^*-\cdots-q_n^*-c)-q_i^* = 0.$$

- That is,

$$\begin{aligned} 2q_1^* + q_2^* + \cdots + q_n^* &= 1-c \\ q_1^* + 2q_2^* + \cdots + q_n^* &= 1-c \\ &\vdots \\ q_1^* + q_2^* + \cdots + 2q_n^* &= 1-c \end{aligned}$$

- Therefore, $q_1^* = \ldots = q_n^* = (1-c)/(n+1)$.