

# Lecture 13

## Infinitely Repeated Games

14.12 Game Theory  
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### Road Map

1. Single-deviation principle
2. Infinitely repeated Entry-deterrence game
3. Infinitely repeated Prisoners' Dilemma
4. Folk Theorem
5. Implicit Cartels

## Infinitely repeated Games with observable actions

- $T = \{0, 1, 2, \dots, t, \dots\}$
- $G =$  “stage game” = a finite game
- At each  $t$  in  $T$ ,  $G$  is played, and players remember which actions taken before  $t$ ;
- Payoffs = Discounted sum of payoffs in the stage game.
- Call this game  $G(T)$ .

## Definitions

The *Present Value* of a given payoff stream  $\pi = (\pi_0, \pi_1, \dots, \pi_t, \dots)$  is

$$PV(\pi; \delta) = \pi_0 + \delta\pi_1 + \dots + \delta^t\pi_t + \dots$$

The *Average Value* of a given payoff stream  $\pi$  is

$$(1-\delta)PV(\pi; \delta) = (1-\delta)(\pi_0 + \delta\pi_1 + \dots + \delta^t\pi_t + \dots)$$

The *Present Value* of a given payoff stream  $\pi$  *at*  $t$  is

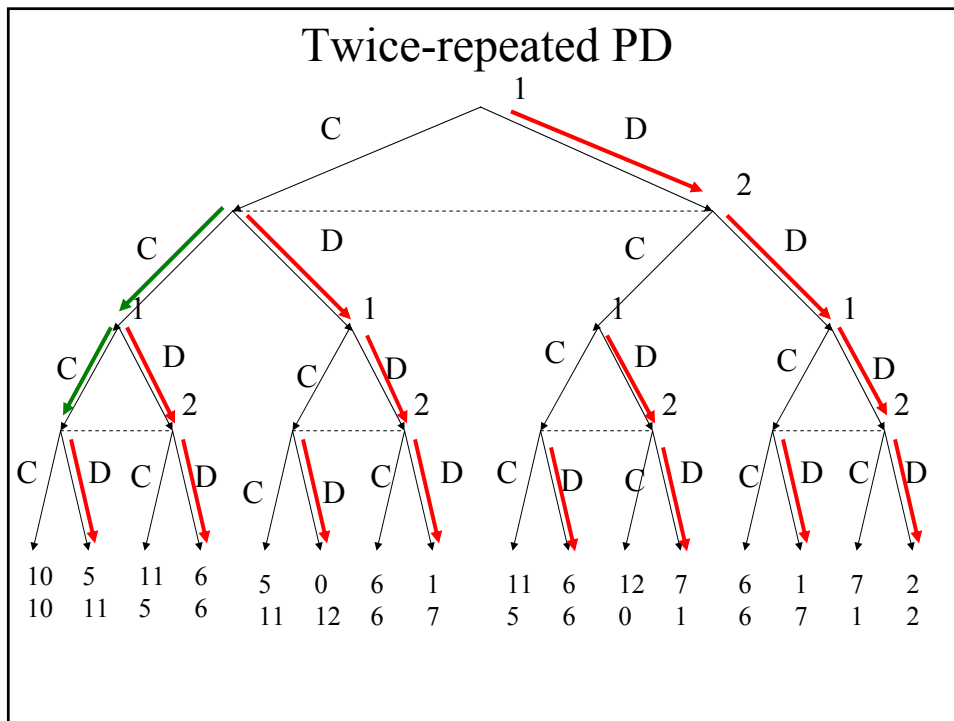
$$PV_t(\pi; \delta) = \pi_t + \delta\pi_{t+1} + \dots + \delta^s\pi_{t+s} + \dots$$

A *history* is a sequence of past observed plays

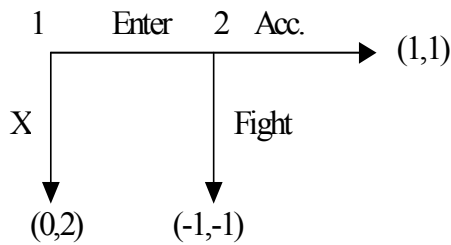
e.g. (C,D), (C,C), (D,D), (D,D) (C,C)

## Single-deviation principle

- $s = (s_1, s_2, \dots, s_n)$  is a SPE
- $\Leftrightarrow$  it passes the following test
- At each history and for each player  $i$ , assume
  - **everybody** will follow  $s$  from tomorrow on,
  - everybody except for  $i$  plays according to  $s$  **today**;
 then  $i$  does not have an incentive to deviate today.



## Infinite-period entry deterrence



### Strategy of Entrant:

Enter iff  
Accommodated  
before.

### Strategy of Incumbent:

Accommodate iff  
accommodated  
before.

#### Incumbent:

- $V(\text{Acc.}) = V_A = 1/(1-\delta)$ ;
- $V(\text{Fight}) = V_F = 2/(1-\delta)$ ;
- Case 1: Accommodated before.
  - Fight  $\Rightarrow -1 + \delta V_A$
  - Acc.  $\Rightarrow 1 + \delta V_A$ .
- Case 2: Not Accommodated
  - Fight  $\Rightarrow -1 + \delta V_F$
  - Acc.  $\Rightarrow 1 + \delta V_A$
  - Fight  $\Leftrightarrow -1 + \delta V_F \geq 1 + \delta V_A$
  - $\Leftrightarrow V_F - V_A = 1/(1-\delta) \geq 2/\delta$
  - $\Leftrightarrow \delta \geq 2/3$ .

#### Entrant:

- Accommodated
  - Enter  $\Rightarrow 1 + \delta V_{AE}$
  - X  $\Rightarrow 0 + \delta V_{AE}$
- Not Acc.
  - Enter  $\Rightarrow -1 + \delta V_{FE}$
  - X  $\Rightarrow 0 + \delta V_{FE}$

## Infinitely-repeated PD

	C	D
C	5,5	0,6
D	6,0	1,1

**A Grimm Strategy:**  
Defect iff someone  
defected before.

- $V_D = 1/(1-\delta)$ ;
- $V_C = 5/(1-\delta) = 5V_D$ ;
- Defected before (easy)
- Not defected
  - D  $\Rightarrow 6 + \delta V_D$ .
  - C  $\Rightarrow 5 + \delta V_C$ .
  - C  $\Leftrightarrow \delta \geq 1/5$ .