

14.27 Problem Set #5

Due on the day of lecture on Portals and Search Engines.

1. Suppose that there are two risky assets in an economy. The return r_1 on stock in company 1 is distributed normally with mean 0.1 and variance 0.05. The return r_2 on stock in company 2 is distributed normally with mean 0.2 and variance 0.25. Assume that the two returns are independent. These are the only two assets in the economy. There is no risk free investment and there is also no cash or physical assets that consumers can use to carry wealth over from one period to the next.

(a) Suppose consumer invests w_1 in stock 1 and w_2 in stock 2 in the first period. His second period wealth W is a random variable given by $W = w_1 (1 + r_1) + w_2 (1 + r_2)$. What is the distribution of W ?

(b) Suppose that the investor has wealth W_0 to invest in the first period. Suppose that he is extremely risk averse and simply wants to minimize the variance of his return. What portfolio does he choose? What is his expected return? What is the variance of his return?

(c) Let \underline{v} be the minimum variance you found in part (b). Note that for any higher level of variance v that the investor is willing to hold he can earn a higher expected return. Assume that the investor is allowed to take a short position either stock, e.g. to choose $w_1 < 0$, and can thereby earn any expected return he wants. Find the maximum possible return for a given $v > \underline{v}$. Graph the expected return as a function of v . (This graph is sometimes referred to as the efficient portfolio frontier.) Put points corresponding to the two stocks on this graph. Why is one point on the graph and one point below it?

(d) Suppose the investor's utility function for second period wealth is $u(w) = -e^{-Aw}$ with $A=3$. Suppose the investor has an initial wealth of \$100,000. How will the investor divide his portfolio between the two assets? (Recall that I noted in class that when W is normally distributed $E(e^{tw}) = \exp(tE(w) + t^2\text{Var}(w)/2)$).

2. Scott Morton, Zettelmeyer and Silva report that Autobytel customers pay an average of about \$400 less for their cars than other buyers.

(a) A crucial question for the future of referral services is whether prices are lower because consumers used Autobytel or whether they are just lower because the consumers using Autobytel are more savvy and would have gotten lower prices anyway. They note that the savings of Autobytel customers vary by car class. They are much larger for purchasers of pickups than for purchasers of luxury goods. Why might this finding be taken to suggest that much of the price savings is due to web customers being more savvy?

(b) Another reason why the lower prices might not be indicative of savings from using Autobytel is that prices may be lower just because the cars Autobytel users buy are worse in unobserved ways, e.g. they may have worse stereo systems. What does the evidence in the paper on the frequency with which Autobytel customers use dealer financing and buy insurance and service contracts suggest about this possibility?