14.30 Exam 1
Spring 2003

Instructions: This exam is closed-book and closed-notes. You may use a calculator. Please read through the exam first in order to ask clarifying questions and to allocate your time appropriately. In order to receive partial credit in the case of computational errors, please show all work. You have approximately 85 minutes to complete the exam. Good luck!

1. (15 points) Suppose you're producing a reality TV show where an ordinary woman is stranded on a desert island with 20 male celebrities, including a set of twins. She has to choose whom among them she would be willing to go on a date with after the show, but is starting to get fed up with their lying and conniving and is threatening to quit in the middle of the filming. You strike a compromise with her: She can administer lie detector tests to five of them chosen at random. If three or more fail, she can quit the show. What is the probability that both of the twins will be chosen for the lie detector test? What is the probability that she will quit if only a quarter of the male celebrities are liars?

2. (15 points)A Monet expert is given a painting purported to be a lost Monet. He is asked to assess the chances that it is genuine and has the following information: In general, only about 1% of "found" paintings he receives turn out to be genuine. Furthermore, "found" paintings have a different frequency of use of certain pigments than genuine Monets do. In particular, cadmium yellow appears in 20% of "found" paintings but only 10% of genuine ones. Raw umber appears in 80% of "found" paintings and only 40% of genuine ones. Burnt sienna appears in 40% of "found" paintings and 60% of genuine ones. This particular painting uses burnt sienna but not cadmium yellow or raw umber. What is the probability that this particular painting is genuine? Did you have to make any additional assumptions to carry out your calculation?

3. (20 points) Suppose that a woman is pregnant three times and each time she has a 45% chance of having a girl alone, a 45% chance of having a boy alone, and a 10% chance of having twins. Suppose also that each twin would have equal probability of being a girl or a boy. Finally, assume the sexes of all children are independent.

a. What is the probability that after three pregnancies, she will have exactly two girls?

b. What is the above probability conditional on her having four children?

c. Let X be the number of children. What is the PDF of X?

d. Let Y be the number of girls. What is the PDF of Y?

4. (30 points)

a. Find the CDFs of the random variables with the following PDFs:

i)

$$f_X(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$

ii)

$$f_y(y) = \begin{cases} \binom{5}{y} \frac{1}{2}^5 & y = 0, 1, \ldots, 5 \\ 0 & \text{else} \end{cases}$$

iii)

b. Find the PDFs of the random variables with the following CDFs:

i)

$$F_U(u) = \begin{cases} 0 & u \leq 1/2 \\ \frac{1}{9}(u - \frac{1}{2})^2 & 1/2 < u < 7/2 \\ 1 & u \geq 7/2 \end{cases}$$

ii)

$$F_V(v) = \begin{cases} 0 & v < 0 \\ i/10 & \frac{i-1}{10} \leq v < \frac{i}{10}, \quad i = 1, \ldots, 10 \\ 1 & v \geq 1 \end{cases}$$

iii)

5. (20 points) You have a dartboard of radius one foot which has an inner centered square whose sides are one foot long. It is divided into quadrants and has points associated with it as shown. Suppose that when throwing darts, you're just as likely to hit any point as any other point on the board. Let X and Y be the random variables which describe the coordinates of a thrown dart, where the center of the board is $(0, 0)$. (Recall that the equation for a circle centered at $(0, 0)$ of radius r is $x^2 + y^2 = r^2$.)

a) What is the joint PDF of X and Y?

b) Are X and Y independent?

c) What is the probability that a thrown dart will land in the center square?

d) Let W be the number of points a dart throw will give you. What is the CDF of W?

e) **(10 extra credit points)** Let $Z = \max\{|X|, |Y|\}$. What is the PDF of Z? (Hint: The answer to part c) will help you.)