Exam 1

14.30, Fall 2000

Instructions: This exam is closed-notes and closed-book. You may use a calculator. PLEASE READ THROUGH THE EXAM FIRST, in order to ask clarifying questions and to allocate your time appropriately. In order to receive partial credit in the case of computational errors, please show all work. You have approximately 85 minutes in which to complete the exam. Good luck!

1. (15 pts.) Suppose, for a moment, that you have two young daughters who want to collect all eight of the current "Barbie Summer Olympics" Happy Meal toys at McDonald's. (For the uninitiated, these are toys that come free with a child's meal at McDonald's.) (Suppose, also, that you unaware that you can entirely avoid eating large quantities of McDonald's food by buying the complete set of these things on Ebay.)

If each Happy Meal has a probability of 1/8 of containing each toy and the events that two Happy Meals each contain particular toys are independent, what is the probability that four trips to McDonald's (buying two Happy Meals each time) will net you all eight of the toys? How much can you increase the probability if you and your spouse each order a Happy Meal as well?

2. (24 pts.) It is claimed that a vitamin supplement helps mice learn to run a maze more quickly and, in fact, to beat a nontreated mouse 70% of the time (whereas without the supplement, it would only win half of the time). To test this claim, you divide a group of 20 mice randomly into ten pairs, giving one of each pair the vitamin supplement. You run each pair through the maze and find that seven out of ten of the mice treated with the vitamin supplement beat the untreated mice.

a) What is the probability of obtaining this result if the claim is true?

b) What is the probability of obtaining this result if the claim is false (*i.e.*, the supplement has no effect)?

c) Suppose before you ran this experiment, you thought it was equally likely that the claim was true or false. After running the experiment, what is your belief? (I'm asking for an updated probability here, not just a verbal description.)

3. (16 pts.)

$$f_X(x) = \begin{cases} \frac{4}{3}(1-x^3) & \text{if } \ 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

a) Find $P(X < \frac{1}{2})$.

b) Find the CDF of X.

c) Find the value t such that $P(X \le t) = \frac{1}{4}$.

d) Find the mean and median of this random variable.

4. (24 pts.) Please answer each of the following questions and give a **brief** explantion (ten words or so should do it).

a) Is this function a PMF?

b) Is this function a PDF?

c) Is this function a CDF?

d) Is this function joint PDF?

e) Are these two random variables independent?

f) Are these two random variables independent?

5. (21 pts.) Suppose that a point in the xy-plane is chosen at random from the interior of a circle for which the equation is $x^2 + y^2 = 1$; and suppose that the probability that the point will belong to any region inside the circle is proportional to the area of that region.

a) What is the joint PDF of X and Y where $(X,\ Y)$ is the set of random variables describing the coordinates of the point.

b) Are X and Y independent?

c) Let Z denote the random variable representing the distance from the center of the circle to the point. What is its PDF and CDF?