

14.41 PS #2 Answers

1a) $U_F = .998*(100,000 + 5)^{1/4} + .002*(0 + 5)^{1/4} = 17.75044105$
 $U_{BS} = .99*(100,000 + 5)^{1/2} + .01*(0 + 5)^{1/2} = 313.0956756$
 $U_{MS} = .99*100,000 + .01*0 = 99000$

1b) The average cost of fires in Springfield is:

$$.5*.002*100,000 + .5*.01*100,000 = 600$$

so the actuarially fair price will be set at \$600. With insurance the utilities will be:

$$U_F = (100,000 - 600 + 5)^{1/4} = 17.75628297$$

$$U_{BS} = (100,000 - 600 + 5)^{1/2} = 315.2855848$$

$$U_{MS} = 100,000 - 600 = 99400$$

Everyone is better off buying insurance at this price, so the price will hold in equilibrium. (The set of numbers in **bold**).

1c) No one who has insurance will buy the device, since doing so costs them \$5 and doesn't change their premium or their expected utility, since each individual has no effect on the equilibrium price and the insurance is full. (Even if, say, all the Flanders got together and agreed to use the device to lower their premium, when device use is unobservable there will be a free rider problem: each Flanders can benefit from the reduced premium but save \$5 by not buying it themselves, since their individual action won't drive up the price of insurance. But then in equilibrium no Flanders will keep their promise, and the premium won't go down.) So, with no monitoring, there's no way to get insured people to buy the device.

But now we must check whether the new option of going uninsured and buying the device is attractive to anyone. First check Flanders (in general, the low-risk type is most likely to select out of the insurance, so check the low-risk type's outside option first). If Flanders buys the device and goes uninsured, utility will be:

$$U_F = .999*(100,000 + 5 - 5)^{1/4} + .001*(0 + 5 - 5)^{1/4} = 17.76501131$$

which is higher than the utility of being insured. So Flanders will opt out of insurance.

This will change the risk pool buying insurance, so the actuarially fair premium will now be:

$$.01*100,000 = 1000$$

Will Bart still want to buy insurance at this price? Bart's utility with insurance at this price is:

$$U_{BS} = (100,000 - 1000 + 5)^{1/2} = 314.6505999$$

If Bart instead opts out of insurance and buys the device, expected utility will be:

$$U_{BS} = .995*(100,000 + 5 - 5)^{1/2} + .005*(0 + 5 - 5)^{1/2} = 314.6466272$$

which is lower than with insurance, so Bart will continue to buy insurance at the new price.

Will Maggie still want to buy insurance at this price? Maggie's utility with insurance at this price is:

$$U_{MS} = 100,000 - 1000 = 99000$$

If Maggie instead opts out of insurance and buys the device, expected utility will be:

$$U_{MS} = .995*(100,000 - 5) + .005*(0 - 5) = 99500$$

which is higher than with insurance, so Maggie will no longer buy insurance. Note that this does not change the equilibrium premium, because the average risk in the insured population has stayed the same.

So, in equilibrium, insurance will cost \$1000, Bart's descendants will buy insurance and not buy the device, and Flanders' and Maggie's descendants will not buy insurance and will buy the device. (The set of numbers in underline).

- 1d) Average social welfare has increased from **24937.69954** to 24961.29516.

The Flanders clan is better off because with the new technology each can partially self-insure, which is better than having full insurance at a price that, for them, is not actuarially fair.

Maggie's descendants are also better off: although they lose the \$400 subsidy that the Flanders were providing in the pooling equilibrium, they are better off by being unsubsidized but cutting their own risk.

Bart's descendants are worse off because they are no longer subsidized by the low-risk Flanders types, but they do not cut their own risk.

Overall, social welfare improves because the efficiency gains of having $\frac{3}{4}$ of society cut its fire risk in half outweighs the efficiency loss of having $\frac{1}{2}$ of society uninsured even though they're risk-averse and of redistribution to the Bart types,

who are risk-averse but high risk. (Note that the Maggie types don't actually benefit from insurance anyway, since they're risk-neutral—they only benefit from the subsidy that Flanders provides).

But note that this results from the fact the Maggie types' utility is implicitly weighted more heavily than the Bart types', which in turn is weighted more heavily than the Flanders types', because of the forms of the utility functions. If instead we normalized and found the percent change in utility for each type, we would find that the Flanders get a 0.05% increase in utility, the Maggies get a .01% increase, and the Barts get a .2% decrease. On average, then, there is a .023% *decrease* in utility. **Lesson: when there are winners and losers, whether social welfare increases or decreases depends on the way different groups' utilities are weighted.**

- 1e) The Flanders opting out of insurance is *adverse selection*—the low-risk types leave the pooling equilibrium with the high-risk types, so those who select to continue being insured are those at higher risk for the adverse event.

The Barts choosing not to buy the device because they can't capture the gains from it, since they're fully insured, is *moral hazard*. If there were no insurance available, the Barts would buy the device and the risk of fire in society would be lower. Having insurance increases the hazard (probability) of the adverse event occurring, relative to what the hazard would be if there were no insurance.

- 1f) If everyone buys the device, the average cost of fires in Springfield is:

$$.5 \cdot .001 \cdot 100,000 + .5 \cdot .005 \cdot 100,000 = 300$$

So the actuarially fair premium is \$300. At that price, utilities will be:

$$U_F = (100,000 - 300 + 5 - 5)^{1/4} = 17.76944197$$

$$U_{BS} = (100,000 - 300 + 5 - 5)^{1/2} = 315.7530681$$

$$U_{MS} = 100,000 - 300 - 5 = 99695$$

Everyone's utility is higher here than if going without insurance, so everyone will buy insurance and the equilibrium price of insurance will be \$300. (The numbers in *italics*.) Average social welfare has increased from 24961.29516 to 25011.57299. Note that here, since everyone is better off, social welfare increases no matter how utilities are weighted—this is a Pareto improvement.

- 1g) Since Burns can enforce the use of the device, there is no moral hazard for the insured—they undertake the same actions they would if they weren't insured. Therefore the risk of fire decreases, making society better off. In addition, the Flanders now get insurance that they value because they're risk-averse, so there's no more adverse selection. And the Barts—the high-risk, risk-averse types—now get to pool their risk with lower-risk types, making them better off. The Maggies

don't value insurance per se, since they're risk neutral, but they do value pooling their risk with lower-risk types, since it increases their expected income—the Maggies are gaming the system.

	<u>Premium</u>	<u>U(Flanders)</u>	<u>U(Bart)</u>	<u>U(Maggie)</u>	<u>U(Social Welfare)</u>
no insurance, no device	n/a	17.75044105	313.0956756	99000	24837.14914
insurance, no device					
pooling	600	17.75628297	315.2855848	99400	24937.69954
partially separating	1000		314.6505999	99000	
separating	<u>1000</u>		<u>314.6505999</u>		<u>24961.29516</u>
no insurance, device	n/a	<u>17.76501131</u>	314.6466272	<u>99495</u>	
insurance, required device	300	17.76944197	315.7530681	99695	25011.57299

2ai) Expected utility of seller 1: $1/2*(\sqrt{64}) + 1/2*(\sqrt{0}) = 4$
 Expected utility of seller 2: $1/2*(\sqrt{0}) + 1/2*(\sqrt{64}) = 4$

(ii) The key here is that utility is maximized when income is smoothed over the two periods. Thus, an arrangement can clearly be struck which will make both sellers better off - the seller which gains from the weather outcome can agree to pay something to the seller that loses. Since there is diminishing marginal utility of income, this will make both parties better off by smoothing their income.

The insurance arrangement which will maximize the total societal utility is that the sellers will agree that whoever wins from the weather the next day will give one-half of his proceeds to the other seller. This will give them each a utility of $\sqrt{32}$, which is 5.66. In total, social welfare will increase from 8 in part (i) to 11.32. You should be able to see that any other division of income leaves you with lower social utility.

(iii) If it is announced that it will be hot, then there will be no market for insurance. This is because the ice cream vendor knows he will get all of the income the next day, so there is no need for him to enter into an insurance arrangement with the hot chocolate salesman. This gives the ice cream vendor an expected utility of 8, and the hot chocolate salesman an expected utility of 0. Total social utility is the same as in (i), but it has fallen from (ii). This is because an efficient insurance market has been destroyed by the introduction of this information.

(iv) Genetic screening can reduce uncertainty about health, and thereby prevent risk pooling—since, if there's certainty about how sick you will become, it's no longer a "risk," just a fact. The redistribution between the badly-off (sick) types and the well-off (healthy) types is then no longer provided by the insurance companies—

information that reduces uncertainty can destroy what had been an welfare-maximizing private market.

- 2bi) Expected utility of seller 1: $1/2 * 1/2 * 64 + 1/2 * 1/2 * 0 = 16$
Expected utility of seller 2: $1/2 * 1/2 * 0 + 1/2 * 1/2 * 64 = 16$
- (ii) There are no gains from the private insurance market here. Any redistribution of income from state A to state B leaves both individuals equally well off. This is because there is no longer diminishing marginal utility of income. With linear utility, there are no gains from insurance. So any insurance arrangement which transfers income will leave society equally well off.
- (iii) For the reasons in (ii), introducing the information about the weather does not affect the total social utility; it remains at 32. The only difference is that now the ice cream vendor has all the money and the hot chocolate vendor has none. Once again, since constant marginal utility meant that the introduction of insurance didn't make us any happier, it means that destroying the insurance market doesn't make us less happy.
- 3a. $U = \ln(c_1) + \ln(c_2)$
- b. $c_2 = (1 + r)(100 - c_1)$
- c. Individuals will max $U = \ln(c_1) + \ln((1 + r)(100 - c_1))$
 $= \ln(c_1) + \ln(1 + r) + \ln(100 - c_1)$
→ FOC: $1/c_1 = 1/(100 - c_1)$
→ $c_1 = 50, c_2 = (1+r)*50$, savings in first period is $100 - c_1$, or 50
- di. This is a fully-funded social security program.
- ii. Private savings will be completely crowded-out by public savings—people want to save 50, and that's exactly what the government is saving for them, so they don't want to do any additional savings.
National savings, on the other hand, won't change—the society is still saving 50 per person from period 1 to period 2. *In general, a fully-funded social security program that doesn't compel people to save more than they wanted to will have no effect on total saving if people are fully rational.*
- iii. There is no effect on social welfare—the same amount is being saved, with the same return, so consumption stays the same in both periods. If people were myopic, however, and weren't saving enough to maximize their lifetime utility, then this program could improve social welfare.
- iv. Then social welfare would improve—people would have the same consumption in the first period, but higher consumption in the second period.
- ei. This is a Pay-As-You-Go (PAYGO), or unfunded, social security program.

- ii. Savings will now fall, because people will receive consumption in the second period in exchange for paying into the system in the first period, but without actually saving money from the first period to the second. *Note that savings will not fall to zero—since the government is now giving them less in the second period than it used to, people will want to re-optimize consumption between the two periods, and therefore will save a bit privately.*
- iii. Since utility is concave in this problem, transferring money from those who are better off (those who haven't lost their savings) to those who are worse off (those who lost their savings) is utility-improving, even though there's no change in the total dollar endowment of society. In particular, since this is log utility, if society didn't transfer to those who had lost their savings, those people would have negative infinite utility, and so total social welfare would be negative infinity.

The winners from this transfer are those who lost their savings, who are made (much) better off due to this transfer. The losers are those who did not lose their savings, since they no longer get the interest on their delayed consumption from period 1 to period 2.

4a)

- (i) Pro: people will have a greater incentive to work longer to replace zeros or low numbers in the calculation. Con: this will have the effect of lowering benefits, since the AIME will be lower for a given amount of work; people who suffer a period of unemployment or women who stay home to care for children will be hurt the most.
- (ii) Pro: rich people don't need Social Security benefits to stay out of poverty. Con: this will discourage people from saving and undermine the political support for Social Security by moving it away from being a universal program.
- (iii) Pro: this will help reduce the rise in number of beneficiaries per covered worker by raising the number of covered workers. Con: we may have a problem later when the next generation retires; can't solve the problem by increasing n indefinitely (just increases the size of the Ponzi game).
- (iv) Pro: Life expectancy has been rising, so it makes sense to increase the number of years one is expected to spend working; also, this may encourage people to work longer. Con: this hurts people who are not able to work in their 60s either because they are injured or because of age discrimination – they receive a lower benefit.

4b)

Pro-privatization:

1. Increased national savings. The money invested in these accounts is actually savings, while there is no savings (or almost none) in the Ponzi game. Higher national savings makes our economy grow faster and raises wages in the long run. And since it would be real saving, it would have return r , rather than just $n+g$.
2. Handles uneven population growth. No problem with generations of different sizes (baby boom vs. baby bust), since each generation saves for its own retirement.
3. Less political risk. No risk of politicians cutting benefits once you reach retirement or dipping into the trust fund, since there is an account in your name.

Anti-privatization:

1. More investment risk. People may make bad investment choices. Even if investment options are restricted, the market may perform badly for one particular generation. Either the government must guarantee some minimal return (which is costly, and will encourage people to take more risk if they are allowed to), or individuals will bear the burden if their investment performs badly, which undermines the “security” point of Social Security.
2. Funding the transition. If we end the Ponzi game, either the last generation that paid in isn’t going to get anything back or current workers will have to fund that generation’s retirement and their own. It’s very costly to end the game.
3. Higher administrative costs. Individual accounts have much higher administrative costs than does the Social Security Administration.
4. Redistribution. The current system redistributes from rich to poor. The new system wouldn’t, so if we want to keep some redistribution, we will have to have another program to do this; the new program may introduce distortions in work or savings, depending on how it is set up, and could be costly.