Lecture 6: The Generalized Heckscher-Ohlin Model

14.581: International Economics I

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Plan of the Lecture

- The 2x2x2 Heckscher-Ohlin model generates very sharp predictions. How do these predictions generalize when we allow for an arbitrary number of factors, goods and countries?
- In this lecture we will:
  1. Develop a general version of the Heckscher-Ohlin model with $I \geq 2$ goods, $J \geq 2$ factors, and $K \geq 2$ countries
  2. Briefly discuss the robustness of the main Heckscher-Ohlin model results to more general environments
  3. Develop a framework for empirically testing this generalized Heckscher-Ohlin model

The General Model: Equilibrium Conditions

- We maintain $A2$, $A3$, $A4$, $A5$ from Lecture 4. We will relax $A1$ and qualify $A6$ (and part of $A3$).
- New notation: $w^k \equiv J \times 1$ vector of factor rewards in country k; $p \equiv I \times 1$ vector of goods prices; $V_j^k \equiv$ endowments of factor j in country k.
- Perfect competition and free trade imply that:

$$p_i \leq c_i \left(w^k \right), \text{ with strict equality if } X_i^k > 0 \text{ for any } i, k \qquad (1)$$

- Factor market clearing requires:

$$\sum_{i \in I} a_{ji} \left(w^k \right) \cdot X_i^k = V_j^k \text{ for any } j, k \qquad (2)$$

- Goods market clearing imposes:

$$\alpha_i (p) = \frac{p_i \sum_{k \in K} X_i^k}{\sum_{i' \in I} \left(p_{i'} \sum_{k \in K} X_{i'}^k \right)} \text{ for any } i \qquad (3)$$

Factor Price Equalization

- In the 2×2 model, we saw that provided that all countries produce all goods and that A is nonsingular at any w^k, equation (1) uniquely pins down w^k in terms of p, which naturally implies FPE.
- In the "even case" in which, $I = J > 2$, this results essentially continues to hold:
  - Nonsingularity of the $J \times I$ technology matrix A is sufficient for local uniqueness of the mapping between w^k and p (inverse function theorem);
  - If additional conditions on the principal minors of A are imposed, *global* univalence holds and we have FPE (Nikaido, 1972).
- When there are less goods than factors ($I < J$), FPE cannot possibly hold since $I \leq rank\,(A) < rank\left(w^k\right) = J$. This implies that factor prices will be sensitive to factor endowments (see Problem Set 1 for the case of the 2×3 specific-factor model).

Factor Price Equalization (cted.)

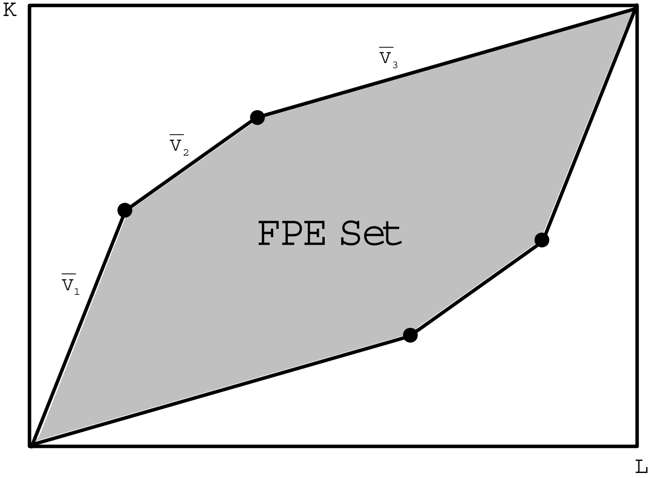
- For the case of (weakly) more goods than factors ($I \geq J$), we can define the *FPE* set as follows:

$$FPE = \left\{ \left(V^1, V^2, ..., V^K\right) \mid \exists \lambda_i^k \geq 0, \sum_{k \in K} \lambda_i^k = 1 \text{ for all } i \right.$$
$$\left. \text{such that } V^k = \sum_{i \in I} \lambda_i^k \bar{V}_i \text{ for all countries } k \right\}$$

  where $\bar{V}_i = \left[a_{1i}(w), .., a_{Ji}(w)\right] \overline{X}_i$ is the employment vector in sector i **in the integrated equilibrium.**

- For the case in which $rank(A) \geq J$, the FPE has positive measure.
- It has been shown that factor price equalization can only hold if $\left(V^1, V^2, ..., V^K\right) \in FPE$. In words, a necessary condition for FPE is that relative factor endowments differences be small relative to relative factor intensity differences (evaluated at integ. eq. w).
- This condition is often referred to as the lens condition (see Deardorff, 1994), but it has not been shown to be generally sufficient (Xiang, 2001, shows it is for $J = 2$).

Factor Price Equalization: An Illustration with 3 goods



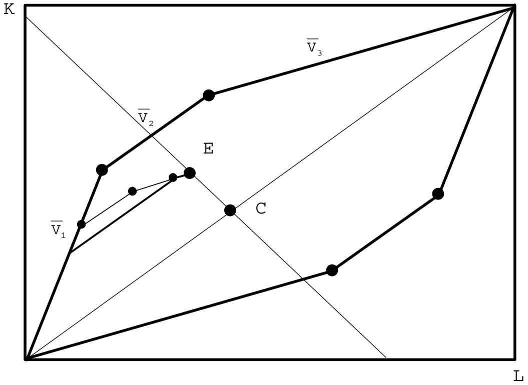
Stolper-Samuelson Theorem

- Log-differentiating the zero-profit condition (assuming diversification), we have
$$\hat{p}_i = \sum_{j \in J} \theta_{ji} \hat{w}^j,$$
where θ_{ji} is factor j's share in the production of i.
- Because $\theta_{ji} \in (0, 1)$, it still has to be the case that there exist two factors j and j' for which $\hat{w}^j \geq \hat{p}_i$ and $\hat{w}^{j'} \leq \hat{p}_i$.
- Jones and Scheinkman (1977) derive more results further illustrating the distributional issues inherent in the Stolper-Samuelson theorem ("friends" vs. "enemies"). Role of factor intensity for $J \geq 2$?
- The following generalization (which applies to goods produced before and after the price change) is perhaps more useful (see Ethier, 1984):
$$\left(w^1 - w^0\right)^T A(\bar{w}) \left(p^1 - p^0\right) \geq 0,$$
for some $\bar{w}$. This suggests that, on average, price increases should lead to increases in the rewards of the factors used intensively in the production of those goods.

Rybczynski Theorem

- Consider first the case in which $I = J > 2$ ("even case") and A is nonsingular. In such a case, we can again invert the A matrix and from (2), we can uniquely pin down the production vector X^k in terms of the endowment vector V^k. Furthermore, it can be shown that an increase in one endowment will necessarily raise the production in one sector and reduce it in another.
- The result does not generalize to the case $I < J$ because even at constant good prices, factor prices (and thus A) will respond to factor endowments. It is then possible to construct examples in which an increase of a factor endowment expands production in all sectors (see Problem Set 1 for an example).
- Finally, the case $I > J \geq 2$ also proves problematic, but for a different reason. In such case, there may be several production vectors X^k that satisfy (2) at a common vector w.
- The theorem can however be restated in terms of correlations: $\left(V^1 - V^0\right)^T A\left(w\right)\left(X^1 - X^0\right) > 0$.

Rybczynski Theorem: An Illustration of the Indeterminacy



- Note, however, that it continues to be the case that the vector of the net factor content of trade is given by EC.

Heckscher-Ohlin Theorem

- The difficulties of the Ryczynski theorem in the cases $I < J$ or $I > J$ carry over to the Heckscher-Ohlin theorem. In the latter case, however, the result can be restated in terms of correlations, or "comvariances" (see Ethier 1984, or Deardorff 1982).
- Sharper results can be obtained in the even case $I = J \geq 2$. In such case, the system of equations in (2) can be inverted and we have (assuming FPE):

$$X^k = A^{-1} V^k.$$

- Subtracting consumption and dividing by country k's share in world income, we obtain

$$X^k - C^k = A^{-1} \left(V^k - s^k \overline{V} \right).$$

- Note that this defines a linear relationship between net exports, factor endowments (V^k) and the technology matrix A^{-1}. This can be taken to the data (see Leamer, 1984; also Harrigan, 1995).

The Heckscher-Ohlin-Vanek Equations

- Suppose that instead of predicting the pattern of commodity trade, we attempted to predict the pattern of net factor content trade.
- Net exports of factor $j \in J$ in country k are given by
$$F_j^k = \sum_{i \in I} a_{ji}(w)\left(X_i^k - C_i^k\right),$$
which we can write in matrix form $F^k = A\left(X^k - C^k\right)$.
- But given homotheticity in preferences, we can write
$$F^k = A\left(X^k - C^k\right) = V^k - s_k \bar{V},$$
and thus $\left(F^k\right)_j > 0$ (i.e., j-th row< 0) if and only if $V_j^k > s_k \bar{V}_j$.
- Furthermore, suppose that we rank factors such that
$$\frac{V_1^k}{\bar{V}_1} > \frac{V_2^k}{\bar{V}_2} > ... > \frac{V_m^k}{\bar{V}_m} > s^k > \frac{V_{m+1}^k}{\bar{V}_{m+1}} > ... > \frac{V_J^k}{\bar{V}_J}.$$
Then we must have $\left(F^k\right)_j > 0$ for all factors $j \leq m$ and $\left(F^k\right)_j < 0$ for all factors $j > m$.

The Heckscher-Ohlin Theorem with No FPE

- We saw that with many goods and FPE, the pattern of trade is in general indeterminate.
- But if factor endowment differences are large enough, FPE may not be attained and factor prices will vary with factor endowments.
- Can the interaction of relative factor intensities and relative factor endowments predict trade in those cases?
- The answer is: "in some situations YES, but in general NO".
- Deardorff (1979) showed that with 2 factors:
  - a "chain of comparative advantage" could be obtained when FPE fails because of differences in relative factor endowments (see also D-F-S, 1980).
  - but in the presence of trade impediments (transport costs, tariffs) *and* intermediate inputs, this chain may break.
  - with many countries, trade impediments are sufficient to break the chain.
- Romalis (2004) obtains sharper predictions in a model with trade frictions, but incorporating monopolistic competition.

Factor Content Predictions without FPE

- In the next lecture we will discuss how to introduce productivity differences (which naturally lead to no FPE) in the H-O-V equations.
- But even for a common technology matrix, factor prices may differ. In such case, the factor content of a country k's gross imports from another country ℓ are given by

$$F_{IMP}^{k,\ell} = \sum_{i \in I} a_{ji} \left(w^{\ell} \right) M_i^{k,\ell},$$

  where $a_i \left(w^{\ell} \right)$ is a $1 \times J$ vector and $M_i^{k,\ell}$ are *gross* imports from k to ℓ.
- We can easily illustrate that not properly adjusting for cross-country differences in factor prices will tend to bias the estimates of the factor content of trade downwards.
  - Suppose two countries H, F and two factors K, L. Suppose Home is capital abundant and imports labor services.
  - Factor price differences naturally lead to $a_{Li} \left(w^H \right) < a_{Li} \left(w^F \right)$ for i.

Factor Content Predictions without FPE (cted.)

- **Actual** labor content of net imports is
$$-F_L^H = \sum_{i\in I} a_{Li}\left(w^F\right) M_i^H - \sum_{i\in I} a_{Li}\left(w^H\right) M_i^F,$$
but this is higher than both
$$\sum_{i\in I} a_{Li}\left(w^H\right) M_i^H - \sum_{i\in I} a_{Li}\left(w^H\right) M_i^F$$
and
$$\sum_{i\in I} a_{Li}\left(w^F\right) M_i^H - \sum_{i\in I} a_{Li}\left(w^F\right) M_i^F,$$
which are the labor contents we would compute with common A.
- Imposing GDP maximization (see Helpman, 1984) or cost min. (see HK, 1985, p. 27), it can also be shown that for any two k and ℓ:
$$\left(w^k - w^\ell\right)\cdot F_{IMP}^{k,\ell} \geq 0 \text{ and } \left(w^k - w^\ell\right)\cdot\left(F_{IMP}^{k,\ell} - F_{IMP}^{\ell,k}\right) \geq 0.$$
- This implies that countries will tend to import those factors (embedded in trade flows) that command high prices in their economy.