14.64 Review Exercise

Labor Economics **Joshua D. Angrist**
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A. A consumer chooses between two goods, x_1 and x_2, with prices p_1 and p_2 given income y, so as to maximize utility.

1. Graph the consumer's problem in x_1 and x_2 space. Show the consumer's optimal choice.

2. Give a graphical decomposition of the income and substitution effects of a change in p_1 on the demand for x_1.

B. Assume the consumer's utility function is given by:

$$u(x_1, x_2) = \alpha \ln(x_1 - \gamma_1) + (1-\alpha)\ln(x_2 - \gamma_2),$$

where: $0 < \alpha < 1$, $x_1 > \gamma_1 > 0$, $x_2 > \gamma_2 > 0$.

(This is called a Stone-Geary utility function. The parameters γ_1 and γ_2 are sometimes thought of as "subsistence" levels of consumption, below which utility is not defined.)

1. What is the marginal rate of substitution (MRS) for this utility function?

2. Present the first-order conditions for optimal choices of x_1 and x_2 as a single equation involving the MRS. What is the graphical interpretation of this condition?

3. Derive the uncompensated ("Marshallian") demand function for x_1 for this consumer as a function of prices and income.

4. Derive the compensated ("Hicksian") demand function for x_1 for this consumer as a function of p_1, p_2 and a fixed level of utility, u^*. Derive the same relationship graphically. What is the connection between the compensated demand function and the substitution effects of question A.2?

C. Applied researchers often work with the Cobb-Douglas production function:

$$Q = \gamma L^{\alpha} K^{\beta}; \; \alpha > 0, \beta > 0, \gamma > 0,$$

where L is Labor, K is capital, and Q is the quantity of output.

1. What must be true for this production function to exhibit constant returns to scale?
2. Econometricians have estimated α and β by fitting the following equation to time series data on measures of output, capital, and labor:

$$LnQ_t = ln\gamma + \alpha lnL_t + \beta lnK_t + \varepsilon_t,$$

where ε_t is an error term and t indexes time series observations. (i) Where does this equation come from? (ii) Assuming you have the necessary data, describe at least two ways to construct a statistical test for constant returns to scale in production.

D. Explain the relationship between the purchase of insurance and the concept of risk aversion. Why might someone who purchases fire insurance also play the lottery?

E. Your uncle gives you a government bond for your Bar Mitzvah that can be given back to the government for 100 dollars in five years. You would rather have cash. How much could you sell the bond for today if the interest rate on FDIC insured savings accounts is fixed at 5 percent (compounded annually)?

F. Suppose that the demand for MP3 downloads is a linear function of their price and the annual income of college students. Make up some notation for a demand equation that expresses this model of demand (assume downloads are not free). Use this equation to compute the price-elasticity of demand for MP3 downloads as a function of price and income.