Graduate Labor Economics Review Exercise

Labor Economics **Angrist, Pischke, Menzel**
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A. A consumer chooses between two goods, x_1 and x_2, with prices p_1 and p_2 given income y, so as to maximize utility.

1. Graph the consumer's problem in x_1 and x_2 space. Show the consumer's optimal choice.

2. Give a graphical decomposition of the income and substitution effects of a change in p_1 on the demand for x_1.

B. Assume the consumer's utility function is given by:

$$u(x_1, x_2) = \alpha \ln(x_1 - \gamma_1) + (1-\alpha)\ln(x_2 - \gamma_2),$$

$$\text{where: } 0 \leq \alpha \leq 1,\ x_1 > \gamma_1 \geq 0,\ x_2 > \gamma_2 \geq 0.$$

(This is called a Stone-Geary utility function. The pameters γ_1 and γ_2 are sometimes though of as "subsistence" levels of consumption, below which utility is not defined.)

1. What is the marginal rate of substitution (MRS) for this utility function?

2. Present the first-order conditions for optimal choices of x_1 and x_2 as a single equation involving the MRS. What is the graphical interpretation of this condition?

3. Derive the uncompensated ("Marshallian") demand function for x_1 for this consumer as a function of p_1 and y by maximization. Use Roy's identity to confirm this.

4. What is the cost function for this problem?

5. Derive the compensated ("Hicksian") demand function for x_1 for this consumer as a function of p_1 and a fixed level of utility, u^* by using Shephard's lemma and by direct solution of the cost minimization problem. What is the graphical interpretation of a compensated price response? What is the slutsky equation for this problem?

6. Show that the consumer's *expenditure* on x_1 is a linear function of p_1 and y. For example, for good x_1, we have

$$p_1x_1 = p_1\gamma_1 + \alpha[y - p_1\gamma_1 - p_2\gamma_2].$$

For this reason, the Stone-Geary utility function is sometimes called the "Linear Expenditure System." Interpret this equation.

C. Empirical researchers often work with the Cobb-Douglas production function:

$$Q = \gamma L^{\alpha} K^{\beta};\ \alpha > 0,\ \beta > 0,\ \gamma > 0,$$

where L is Labor, K is capital, and Q is the quantity of output.

1. What must be true for this production function to exhibit constant returns to scale?

2. Researchers have estimated α and β by fitting the following equation to time series data on measures of output, capital, and labor:

$$LnQ_t = \ln\gamma + \alpha \ln L_t + \beta \ln K_t + \varepsilon_t,$$

where ε_t is an error term and t indexes time series observations. Where does this equation come from? Assuming you have the necessary data, describe at least two ways to construct a statistical test for constant returns to scale in production. Now suppose you have data on a panel of firms. Explain how to use this data to control for an unobserved but time-invariant variable that captures managerial efficiency.

D. People are willing to pay for insurance because they are risk averse. Explain.

E. What is the present value of x dollars/year for t years with interest compounded annually? What is the present value with interest compounded continuously?

F. Suppose that the demand for internet connect time is a linear function of the price of time and the annual income of college students. Make up some notation for a demand equation that reflects this assumption. Use this equation to compute the price-elasticity of demand for connect time as a function of price, income, and quantity sold.

G. What is the relationship between the regression coefficient from a bivariate regression of wages on schooling (a “short regression”) and the coefficient on schooling when the equation also includes test scores (a “long regression”)? Generalize this to the case where both models include other covariates.

H. Explain how to calculate a multivariate regression coefficient using a two-step procedure where the second step is a bivariate regression.

I. Suppose the probability a woman works is modeled using a latent-index model such that

$$y_i = 1[X_i'\beta > \varepsilon_i]$$

where y_i is employment status, X_i is a vector of personal characteristics, and ε_i is a random factor that is assumed to be normally distributed $N(0, \sigma^2)$ and independent of X_i. Describe how the parameters in this model be estimated by maximum likelihood (i.e., write down the likelihood function). What is identified? Propose a weighted least squares procedure that is asymptotically equivalent to the MLE.

J. Derive the formula for the standard error of a difference in means. Show that the optimal experimental design for estimating a difference in means sets the proportion treated at 1/2 if the data are homoscedastic.